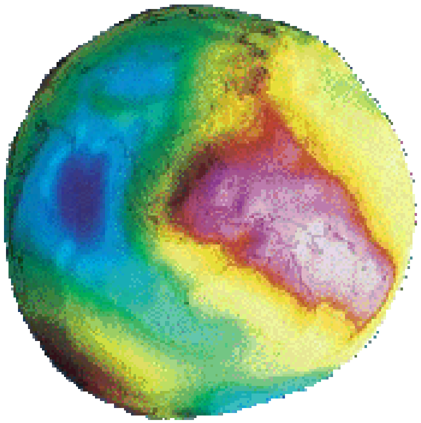
Elements of Geodesy

Shape of the Earth
Tides
Terrestrial coordinate systems
Inertial coordinate systems
Earth orientation parameters



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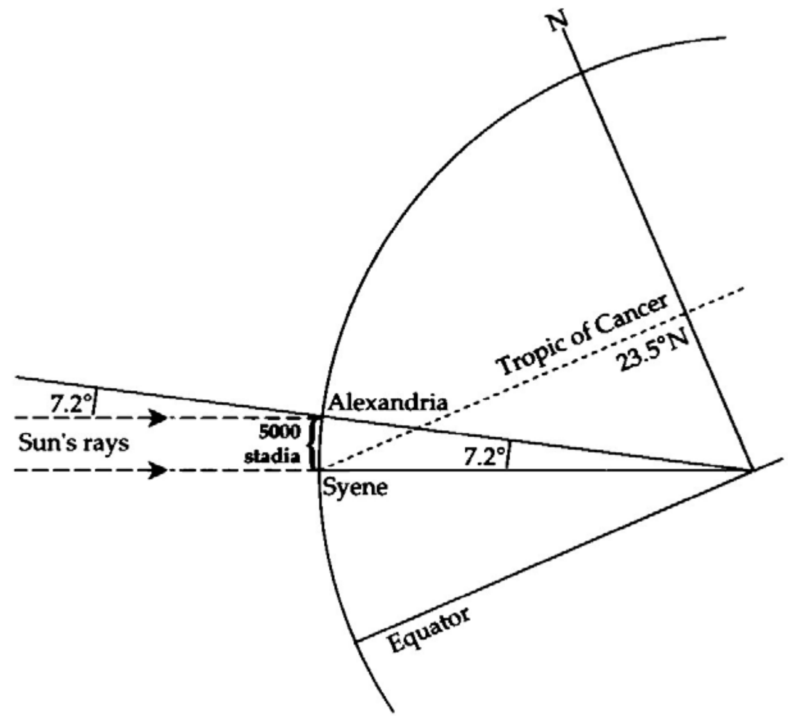
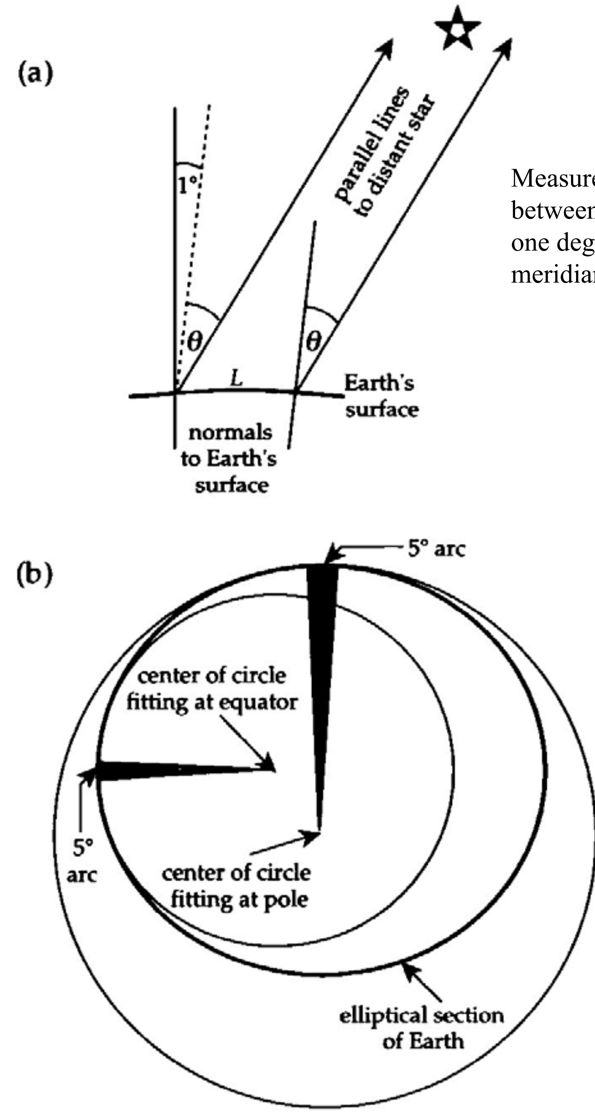
The shape of the Earth/A sphere

- An oyster… (the Babylonians before 3000 B.C.)
- Pythagore (582-507 B.C.): intuition…
- Erathostene (275-195 B.C.), head librarian in Alexandria, Egypt:
  - 2 wells in Assouan (=Syene) and Alexandria
  - Sun rays vertical at Assouan at noon on mid-summer day, but shadow on well side in Alexandria
  - Knowing the Assouan-Alexandria distance = 10 camel days = 5000 stadia
  - 5000 stadia x 185 = 925 000 m: Earth's radius R = 925 000/(7.2 x π/180) = 7 364 650 m
  - Earth circumference = 2 x π x R = 46 250 000 m, 40 030 000 m in reality => 15% error
  - Assouan and Alexandria not exactly on the same meridian
- Spherical earth models often used for short range navigation (VOR-DME), for global distance approximations, in seismology, plate kinematics, etc…: ~14 km difference for polar radius



The shape of the Earth/An ellipsoid

- Newton: Earth rotation + fluid => should be elliptical, computes flattening = 1/230 (1/298 in reality)
- 2 French expeditions for measuring the length of one degree of meridian:
  - Clairaut, 1736, Laponia
  - Bouguer, 1743, Peru
  - Result: 1 degree longer in Laponia than in Peru
  - Sphere tangent to the pole has a larger radius than at the equator, therefore the poles are flattened
  - First definition of the meter: *"10 millionth part of a quarter of the circumference of an Earth's meridian"*

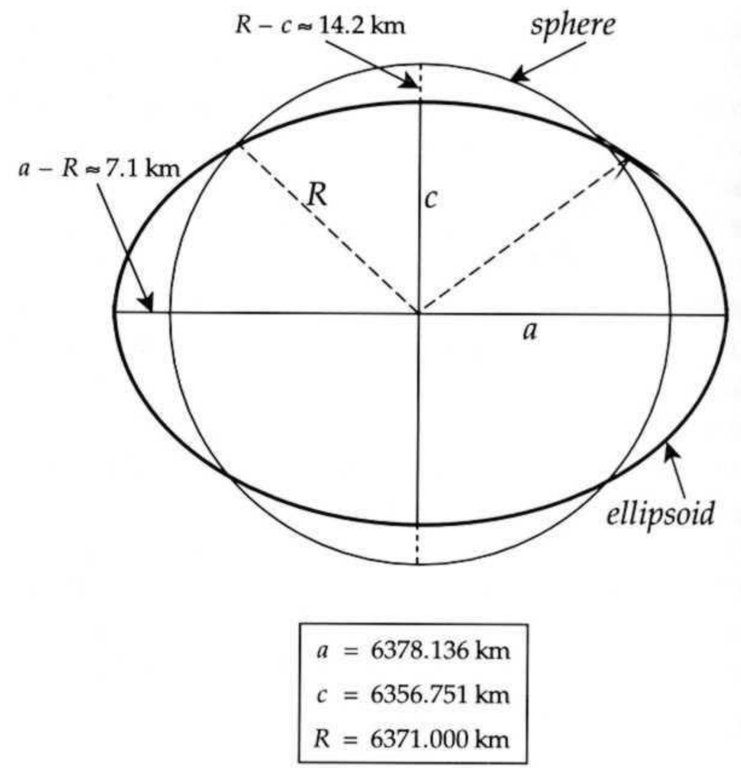


Measurement of the distance between two points that lie one degree apart on the same meridian

The shape of the Earth/An ellipsoid

<u>The shape of the Earth can be mathematically represented as an ellipsoid</u> defined by:

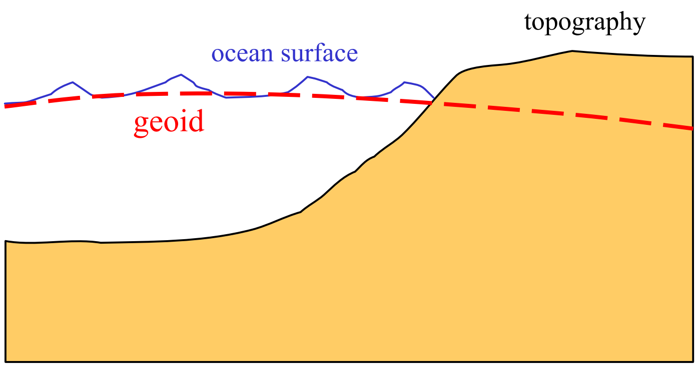
- Semi-major axis = equatorial radius = ***a***
- Semi-minor axis = polar radius = ***c***
- Flattening (the relationship between equatorial and polar radius): $f = (a-c)/a$
- Eccentricity: $e^2 = 2f - f^2$

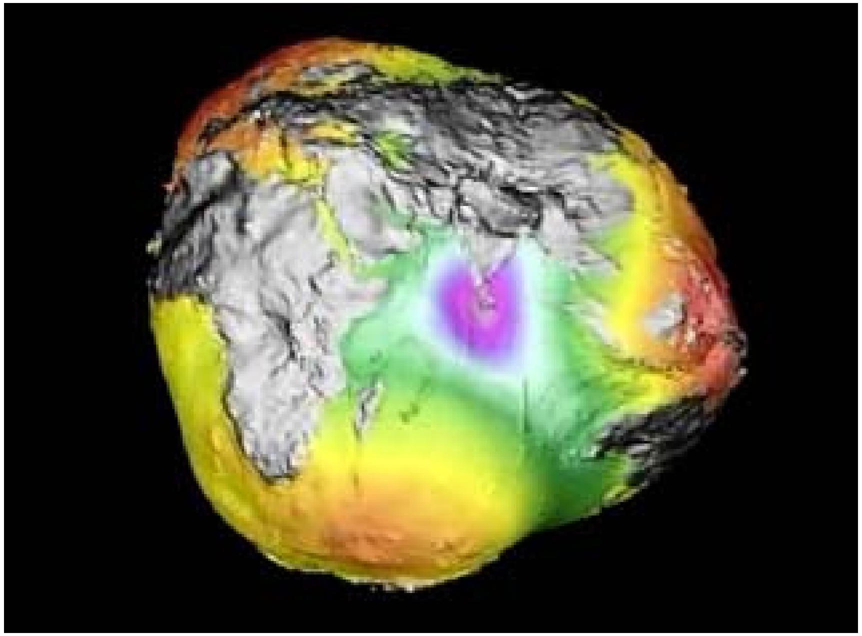


Comparison between the WGS-84 ellipsoid and a sphere of identical volume

The Geoid

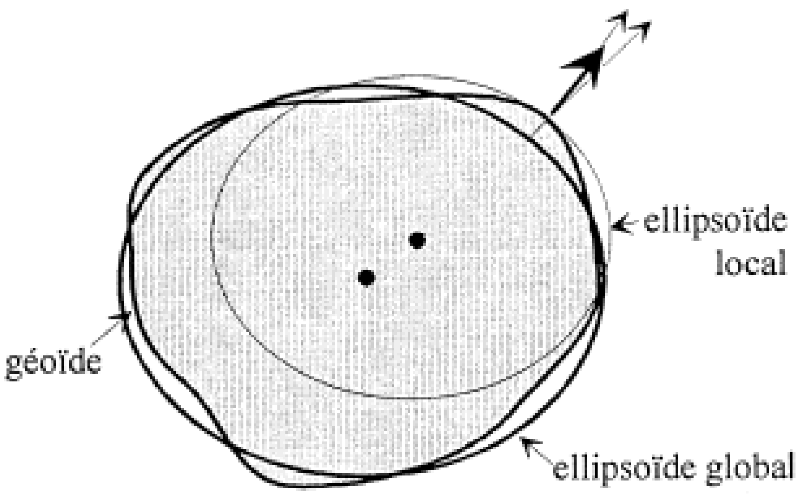
- The Geoid = the particular **equipotential surface that coincides with the mean sea level**
- Over the oceans, the geoid is the ocean surface (assuming no currents, waves, etc)
- Over the continents, the geoid is not the topographic surface (its location can be calculated from gravity measurements)
- Geoid "undulations" are caused by the distribution of mass in the Earth
- Geoid = the "figure" of the Earth (a "potatoid"…?)





Reference ellipsoids

- Many different reference ellipsoids have been defined and are in use!
- WGS-84: best-fit ellipsoid to a smooth averaged Earth surface



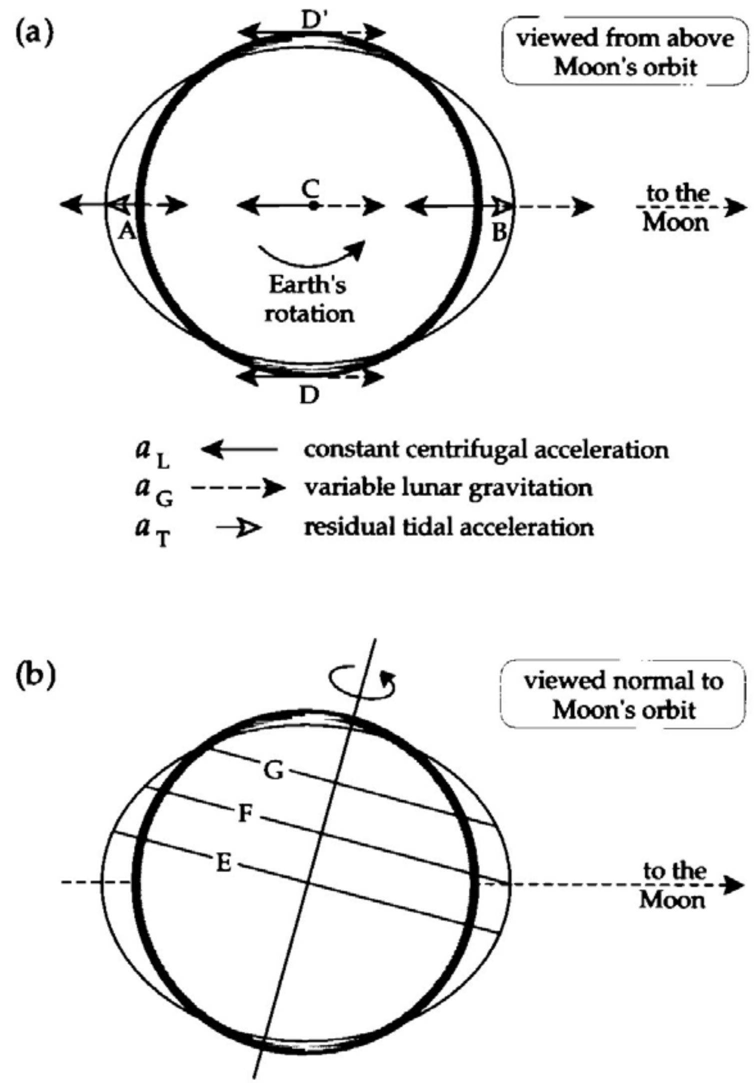
Selected Reference Ellipsoids

Ellipse	Semi-Major Axis (meters)	1/Flattening
Airy 1830	6377563.396	299.3249646
Bessel 1841	6377397.155	299.1528128
Clarke 1866	6378206.4	294.9786982
Clarke 1880	6378249.145	293.465
Everest 1830	6377276.345	300.8017
Fischer 1960 (Mercury)	6378166.0	298.3
Fischer 1968	6378150.0	298.3
G R S 1967	6378160.0	298.247167427
G R S 1975	6378140.0	298.257
G R S 1980	6378137.0	298.257222101
Hough 1956	6378270.0	297.0
International	6378388.0	297.0
Krassovsky 1940	6378245.0	298.3
South American 1969	6378160.0	298.25
WGS 60	6378165.0	298.3
WGS 66	6378145.0	298.25
WGS 72	6378135.0	298.26
WGS 84	6378137.0	298.257223563

Peter H. Dana 9/1/94

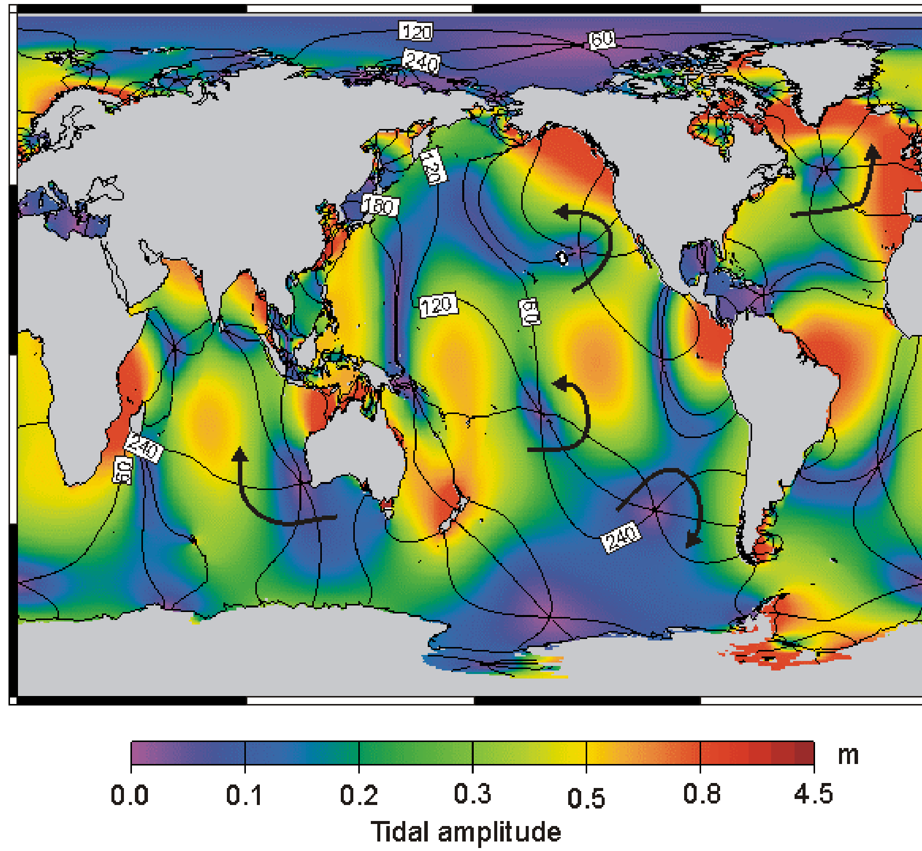
Tides

- **The gravitational forces of the Sun and Moon deform the Earth's shape** => tides in the oceans, atmosphere, and solid earth
- Tidal effect of the Moon:
  - Earth and Moon are coupled by gravitational attraction: each one rotates around the center of mass of the pair.
  - The rotation of the Earth around that center of mass induces a **centrifugal acceleration** directed away from the Moon
  - The Moon produces a **gravitational attraction** on the Earth
  - The resulting force (centrifugal acceleration + gravitational attraction) is responsible for the tides
- Tidal effect of the Sun: same principle but 45% smaller effect because of large Earth-Sun distance



Tides

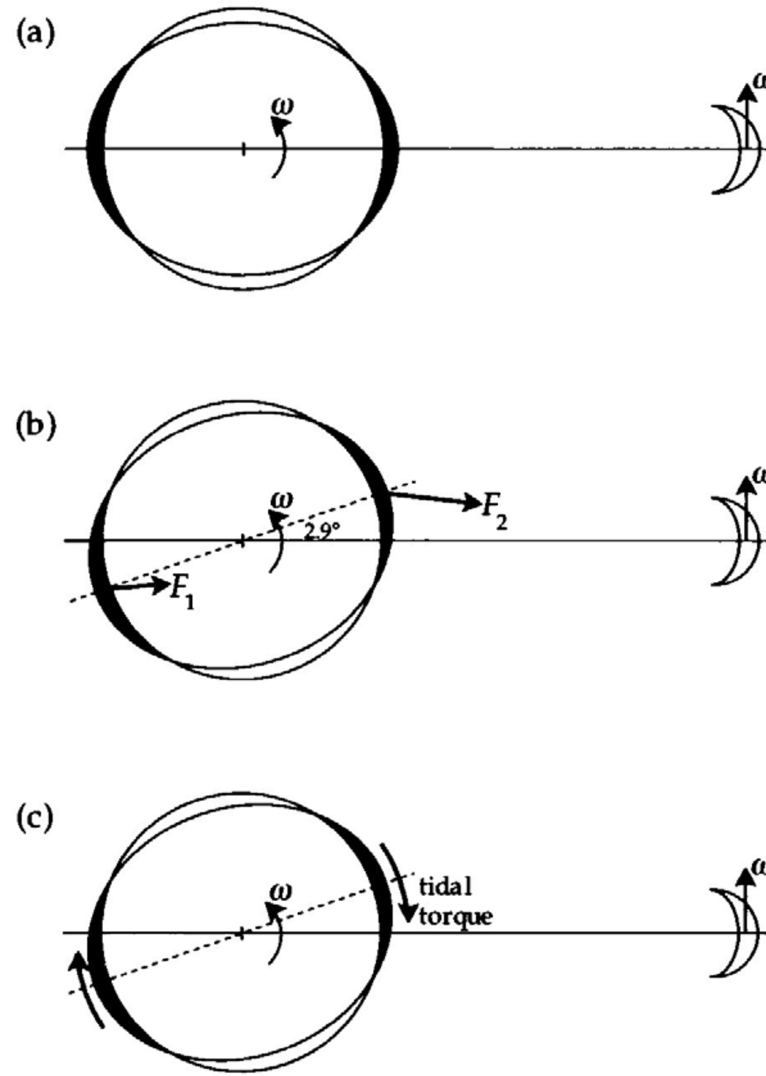
- Earth rotation (24 hr) combined with Moon revolution (~27 days) => major tidal component is semi-diurnal (M_2 = 12 hr 25 min)
- Ocean tides: effect of ocean surface, amplitude of largest component several meters
- Solid Earth tides: effect on the solid Earth surface, amplitude, amplitude of largest component ~10-50 cm
- By-product of ocean tides: ocean tide loading = deformation of the Earth crust due to variations of ocean water column: up to 15-20 cm near the coasts



The ocean tides for harmonic M2 (period of 12 hours and 25 minutes) . The color represent the amplitude and the contour lines indicate the phase lag of the tides with a spacing of 60 degrees. (Doc. H.G. Scherneck)

The shape of the Earth/Tides

- If the Earth was purely elastic => tidal bulge aligned with the Moon (a)
- But Earth tidal response is not instantaneous because of Earth anelasticity => slight delay between high tide and Moon alignment (12 min)
- This creates a torque that tends to bring the the tidal bulge axis back into the Earth-Moon direction, in a direction opposite to the Earth's rotation => deceleration of the Earth's rotation => increase of length-of-day
  - Analysis of growth rings of fossil corals, 350 Ma old => 1 day = 22 hr, 1 year = 400 days
- The Earth tidal bulge creates a similar torque on the Moon, in opposite direction (conservation of Earth-Moon angular momentum) => deceleration of the Moon revolution
- Because of Kepler's third Law ($period^2/a^3$=const), the Moon-Earth distance increases (3.7 cm/yr)



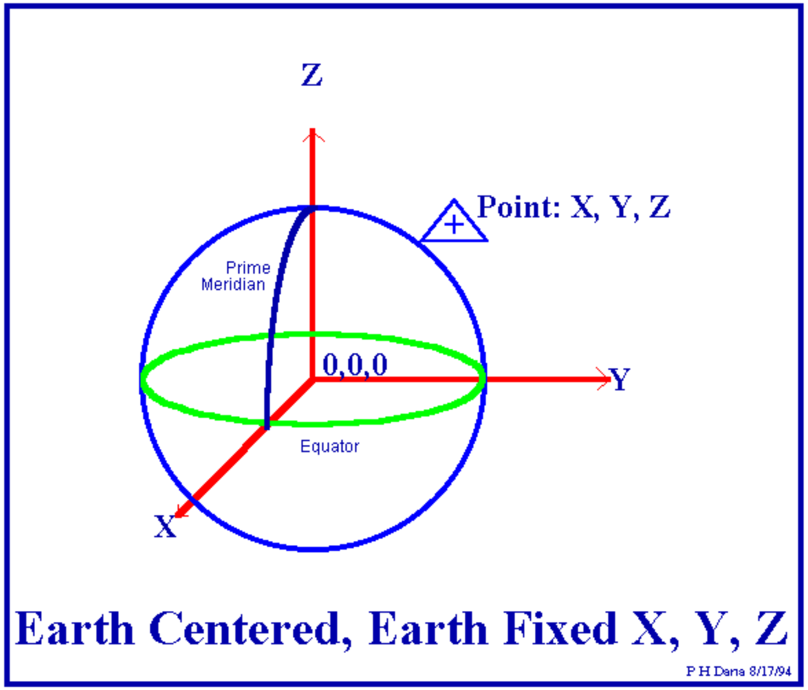
Coordinate systems

- **A coordinate system is defined by:**
  - **It origin** = 3 parameters
  - **Its orientation** = 3 parameters, usually direction of 3 axis
  - **Its scale** = 1 parameter
- 7 parameters are needed to uniquely define a coordinate system
- 7 parameters are needed to go from a coordinate system to another

Coordinate systems

The simplest datum is an **Earth Centered, Earth Fixed cartesian datum** (ECEF):

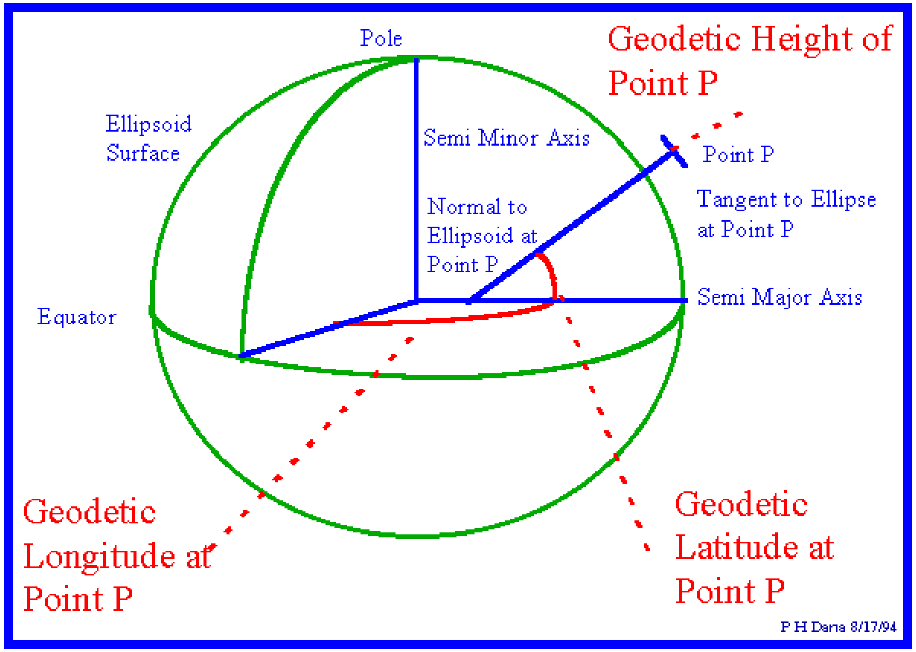
- Origin = center of mass of the Earth's.
- Three right-handed orthogonal axis X, Y, Z.
- Units are meters.
- The Z axis coicincides with the Earth's rotation axis.
- The (X,Y) plane coincides with the equatorial plane.
- The (X,Z) plane contains the Earth's rotation axis and the prime meridian.



Coordinate systems

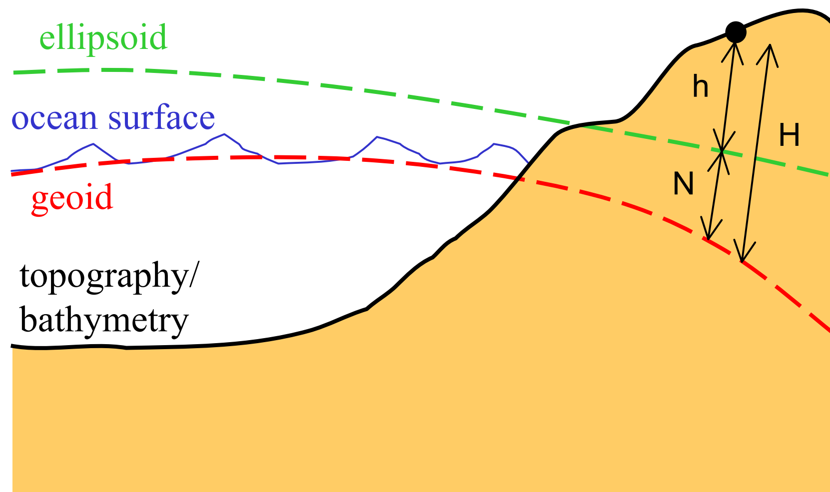
Given that the shape of the Earth is close to an ellipsoid, it is convenient to define a position by its latitude, longitude, and height = **ellipsoidal coordinates**:

- The Prime Meridian is the origin for longitudes. The Equator is the origin for latitudes.
- **Geodetic latitude** = angle from the equatorial plane to the vertical direction of a line normal to the reference ellipsoid.
- **Geodetic longitude** = angle between a reference plane and a plane passing through the point, both planes being perpendicular to the equatorial plane.
- **Geodetic height** = distance from the reference ellipsoid to the point in a direction normal to the ellipsoid.



Coordinate systems

- Conventionally heights are measured above an equipotential surface corresponding to mean sea level (MSL) = the geoid
- Topographic maps, markers, etc.: height above or below **mean sea level = orthometric height**.
- Height above or below a reference ellipsoid (e.g. from GPS reading) = **ellipsoidal height**.
- Difference between the two = **geoid height**
- Transformation from ellipsoidal height to orthometric height requires to know the geoid height.



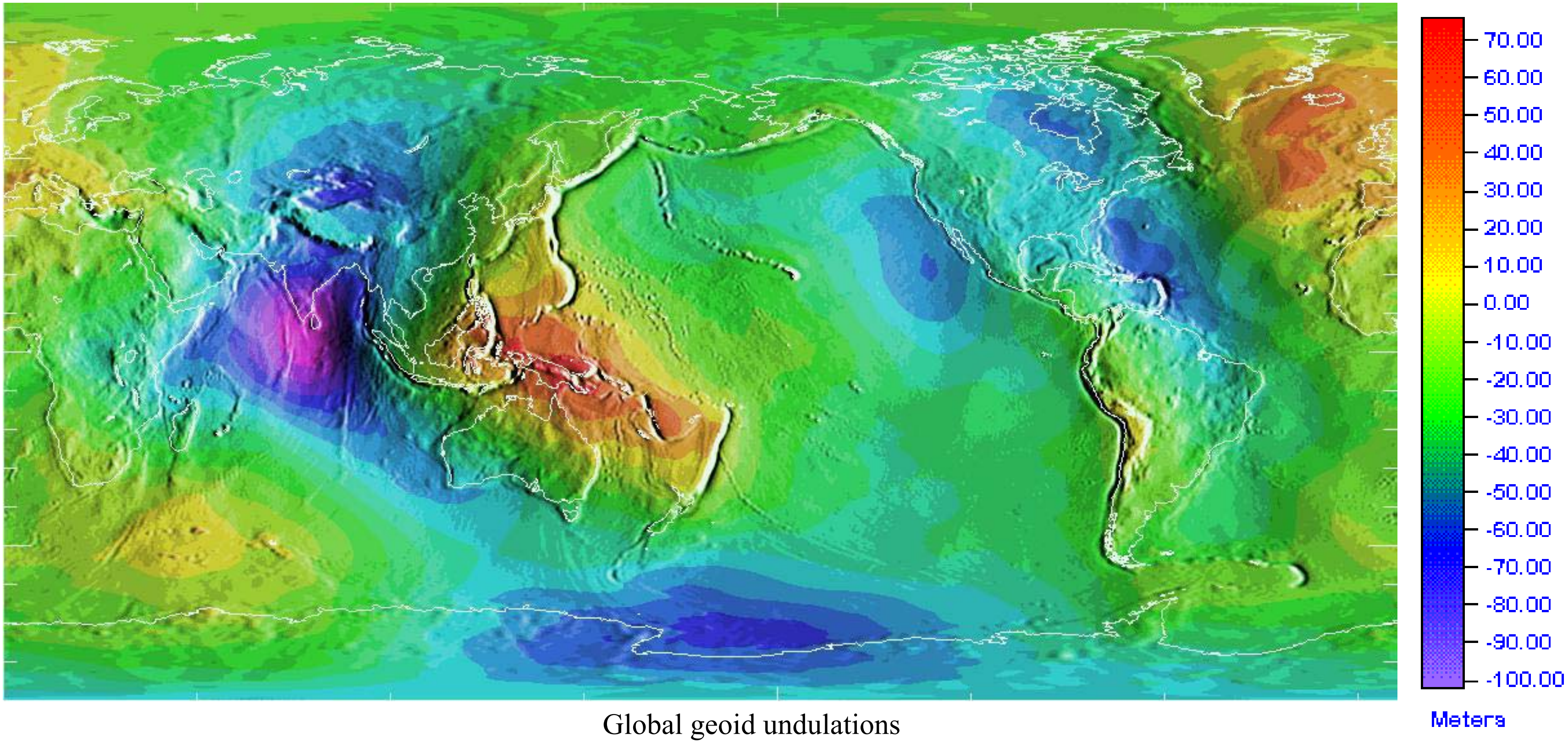
$$\Rightarrow H = h + N$$

h = ellipsoidal height
N = geoid height

H = orthometric height
= height above mean sea level

Coordinate systems

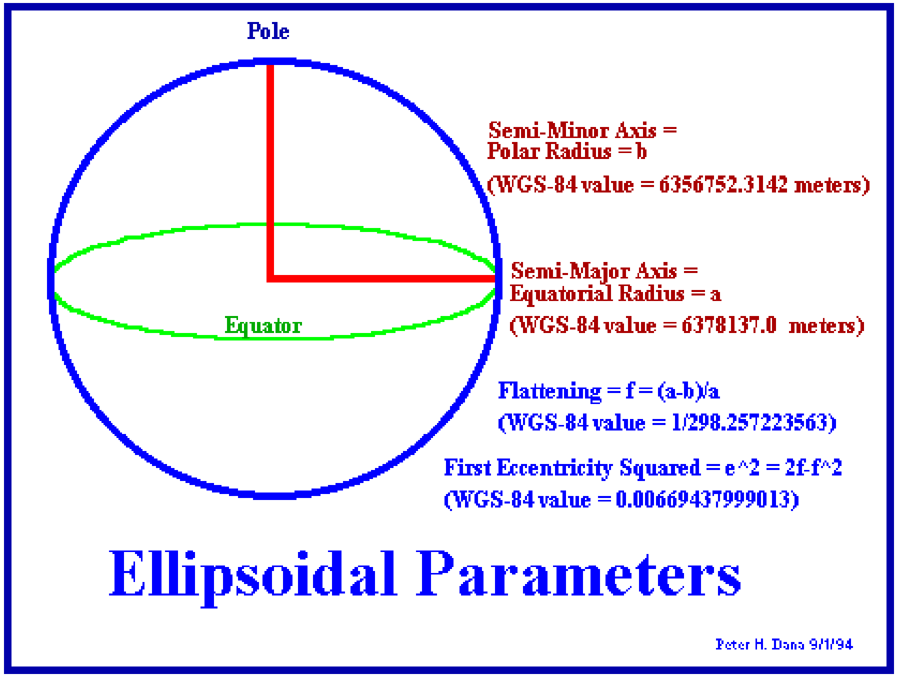
- Geoid height varies (globally) within +- 100 m
- In the conterminous US, geoid height varies from 51.6 m (Atlantic) to 7.2 m (Rocky Mountains)



Global geoid undulations

Coordinate systems

- Note that the latitude, longitude, and height of a given point will be different if different **datums** are used…!
- Datum =
  - The size and shape of the Earth, usually approximated as an ellipsoid: semi-major axis *a* and flattening *f*.
  - The translation of its origin with respect to the Earth's center of mass (t_X, t_Y, t_Z)



Coordinate systems

- Many different reference ellipsoids have been defined and are in use!
- **<u>Reference ellipsoid = the ellipsoid that best fits the geoid</u>**.
- Totally arbitrary, but practical
- Reference ellipsoid = WGS-84
- Geoid undulations = differences, in meters, between the geoid reference ellipsoid (= geoid "height").

Selected Reference Ellipsoids

Ellipse	Semi-Major Axis (meters)	1/Flattening
Airy 1830	6377563.396	299.3249646
Bessel 1841	6377397.155	299.1528128
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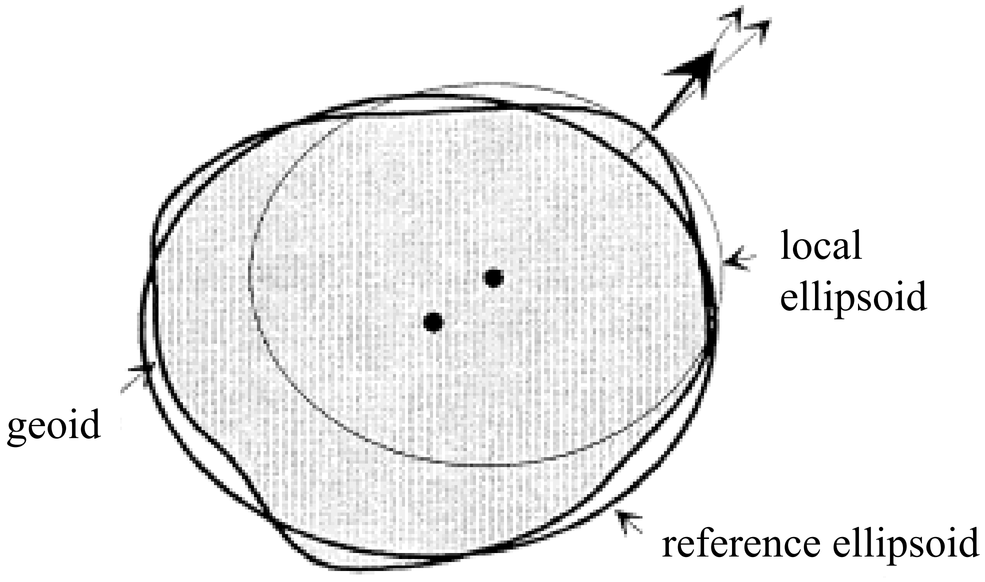
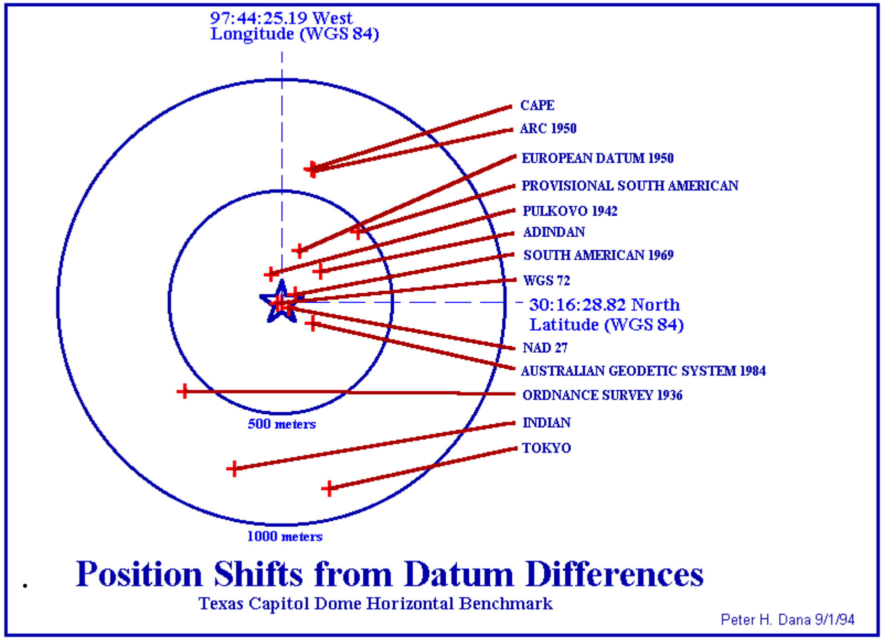


Coordinate systems

- Different countries, agencies, and applications use different datums.
- Hundreds of geodetic datums are in use around the world. The Global Positioning system is based on the World Geodetic System 1984 (WGS-84).
- Using an incorrect datum to express coordinates can result in position errors of hundreds of meters.
- Great care must be used when manipulating coordinates to ensure that they are associated with a well-defined datum.
- Datums are derived from measurements, from classical triangulation surveys to space-based techniques. Therefore, they evolve as the precision and capabilities of the observation techniques improve.



Coordinate systems

Conversion between ECEF and ellipsoidal coordinates can be made using the following formulas:

Coordinate Conversion
Geodetic Latitude, Longitude, and Height to ECEF, X, Y, Z

$$X = (N + h)\cos\phi\cos\lambda$$
$$Y = (N + h)\cos\phi\sin\lambda$$
$$Z = [N(1 - e^2) + h]\sin\phi$$

where:

ϕ, λ, h = geodetic latitude, longitude, and height above ellipsoid

X, Y, Z = Earth Centered Earth Fixed Cartesian Coordinates

and:

$N(\phi) = a / \sqrt{1 - e^2 \sin^2\phi}$ = radius of curvature in prime vertical

a = semi-major earth axis (ellipsoid equatorial radius)

b = semi-minor earth axis (ellipsoid polar radius)

$f = \frac{a - b}{a}$ = flattening

$e^2 = 2f - f^2$ = eccentricity squared

Peter H. Dana 8/3/96

Coordinate Conversion: Cartesian (ECEF X, Y, Z) and Geodetic (Latitude, Longitude, and Height)

Direct Solution for Latitude, Longitude, and Height from X, Y, Z

This conversion is not exact and provides centimeter accuracy for heights < 1,000 km (See Bowring, B. 1976. Transformation from spatial to geographical coordinates. Survey Review, XXIII: pg. 323-327)

$$\phi = \operatorname{atan}\left(\frac{Z + e'^2 b \sin^3\theta}{p - e^2 a \cos^3\theta}\right)$$
$$\lambda = \operatorname{atan2}(Y, X)$$
$$h = \frac{p}{\cos(\phi)} - N(\phi)$$

where:

ϕ, λ, h = geodetic latitude, longitude, and height above ellipsoid

X, Y, Z = Earth Centered Earth Fixed Cartesian coordinates

and:

$p = \sqrt{X^2 + Y^2}$ $\quad \theta = \operatorname{atan}\left(\frac{Za}{pb}\right)$ $\quad e'^2 = \frac{a^2 - b^2}{b^2}$

$N(\phi) = a / \sqrt{1 - e^2 \sin^2\phi}$ = radius of curvature in prime vertical

a = semi-major earth axis (ellipsoid equatorial radius)

b = semi-minor earth axis (ellipsoid polar radius)

$f = \frac{a - b}{a}$ = flattening

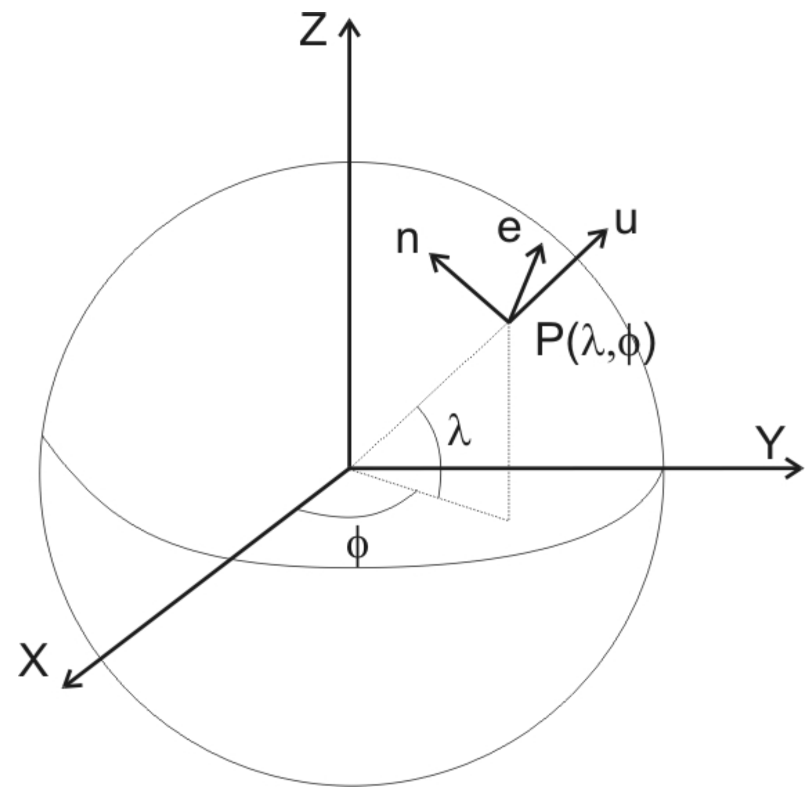
$e^2 = 2f - f^2$ = eccentricity squared

Coordinate systems

- Coordinates can also be refering to a local **topocentric datum**. The origin of this datum is any point you choose on the surface of the Earth. It has 3 right-handed orthogonal axis: u (for “up”) is vertical (= perpendicular to the local equipotential surface) and points upwards, n (for “north”) is in the local horizontal plane and points to the geographic north, e (for “east”) is in the local horizontal plane and points to the geographic east. Units are meters.
- The conversion is a combination of 3 rotations needed to align the ECEF axis with the NEU axis:

$$\begin{bmatrix} n \\ e \\ u \end{bmatrix} = \begin{bmatrix} -\sin\lambda\cos\phi & -\sin\lambda\sin\phi & \cos\lambda \\ -\sin\phi & \cos\phi & 0 \\ \cos\lambda\cos\phi & \cos\lambda\sin\phi & \sin\lambda \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

- where *[X,Y,Z]* is the vector to be transformed (in meters), and λ and ϕ the latitude and longitude of the reference point, respectively.



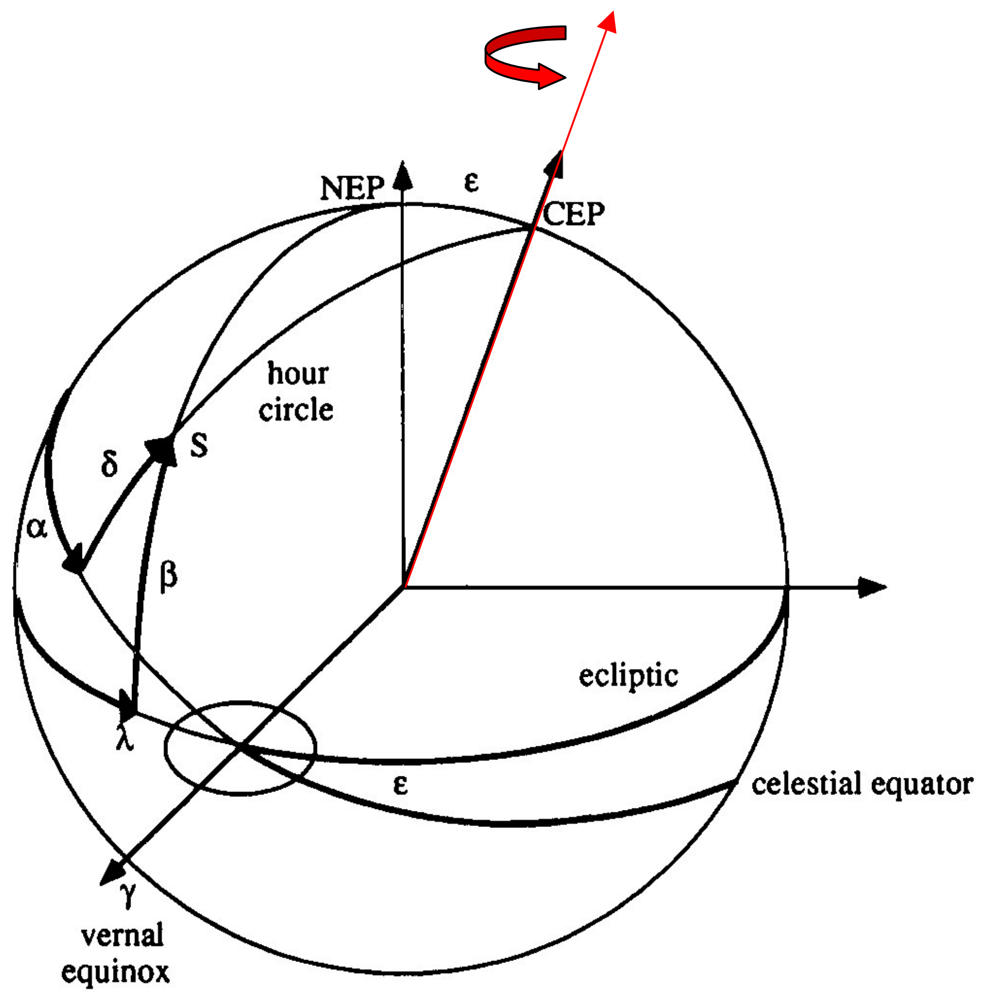
Coordinate systems

See also:

- http://www.colorado.edu/geography/gcraft/notes/datum/datum.html
- http://www.hdic.jmu.edu/sic/content/std_datum.html
- http://www.nima.mil/GandG/tm83581/toc.htm
- http://www.lsgi.polyu.edu.hk/cyber-class/geodesy/syllabus.htm
- http://www.uz.ac.zw/engineering/GeoInformatics/Notes/GEODESY/Geodesy1.htm

Space-fixed coordinate system

- The coordinate system:
  - 2 fundamental planes: celestial equator and ecliptic (= Earth's orbital plane) (angle ε = obliquity)
  - Their poles: CEP (celestial ephemeris pole) and NEP (north ecliptic pole)
  - Their intersection: vernal equinox
  - Earth's rotation axis coincides with CEP (actually angular momentum axis)
  - The center: Earth's center of mass
- Coordinates of an object (e.g. star) uniquely defined by:
  - Right ascension α = angle in equatorial plane measured CW from the vernal equinox
  - Declination δ = angle above or below the equatorial plane
- Or by:
  - Ecliptic longitude λ
  - Ecliptic latitude β
- (α,δ) and (λ,β) related through ε

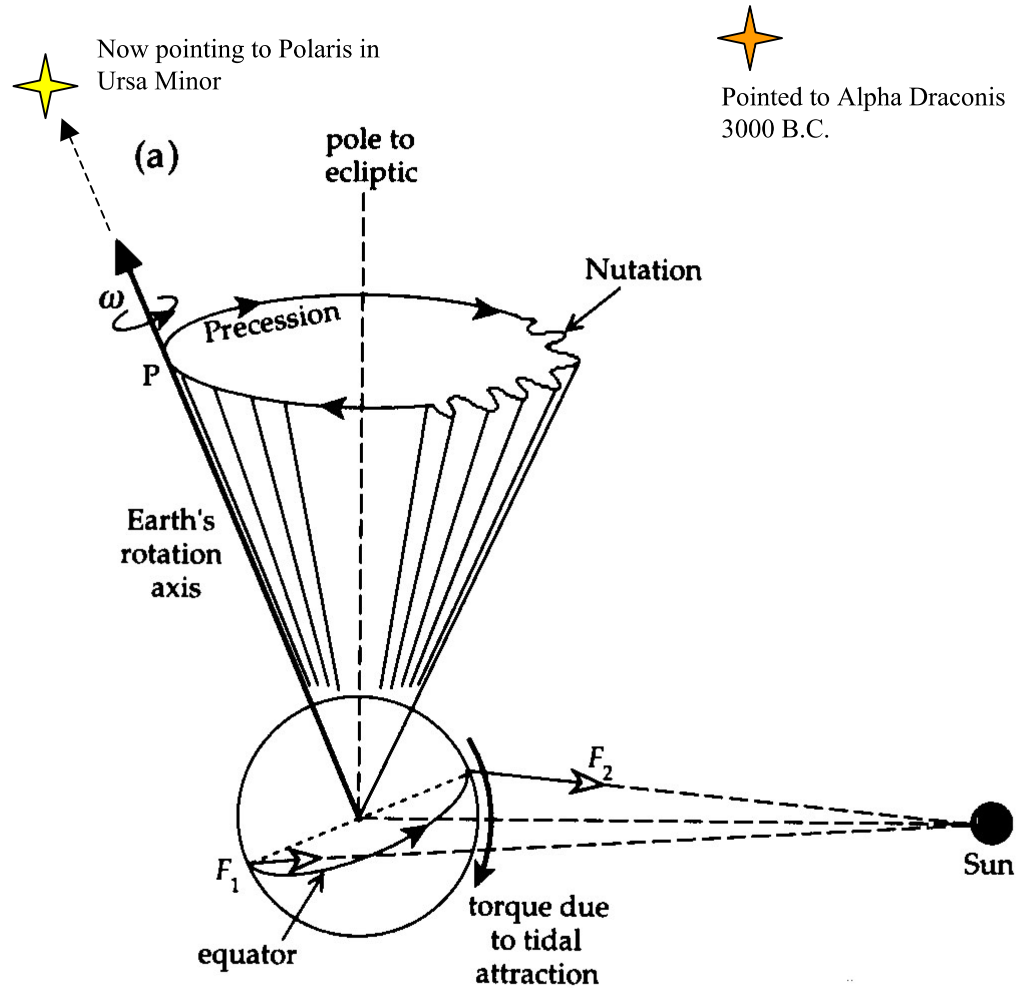


Space-fixed coordinate system/ Oscillation of axes

- Earth rotation vector ω_E: $\vec{\omega}_E = \vec{\omega}\|\vec{\omega}_e\|$
- ω_E oscillates because of:
  - Gravitational torque exerted by the Moon, Sun and planets
  - Displacements of matter in different parts of the planet and other excitation mechanisms
- Oscillations of the Earth rotational vector:
  - Oscillations of unit vector ω:
    - With respect to inertial space (= stars), because of luni-solar tides: **precession** and **nutation**
    - With respect to Earth's crust: **polar motion**
  - Oscillations of norm = variations of speed of rotation = variations of time

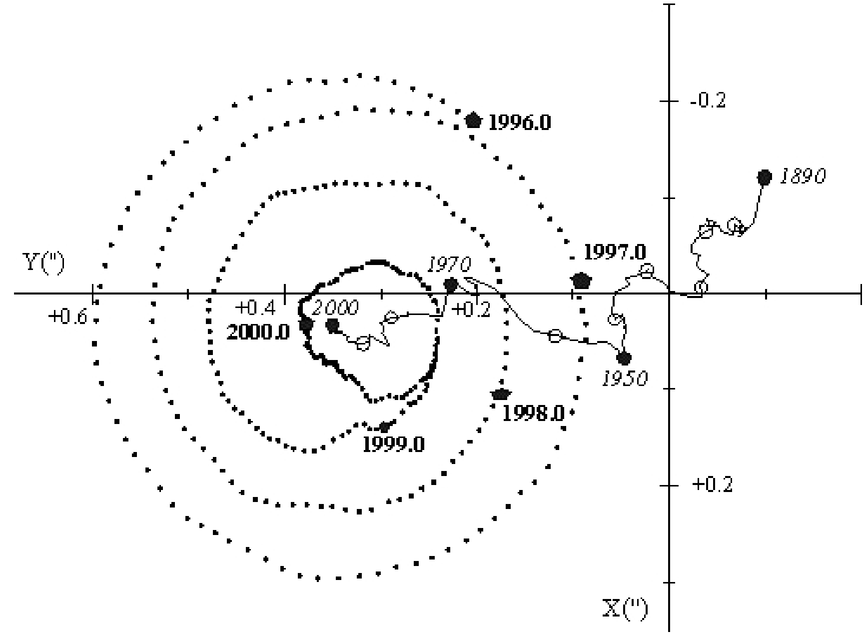
Space-fixed coordinate system/ Precession-nutation

- Because of luni-solar tides, the earth rotation axis oscillates with respect to inertial space (= stars)
- This oscillation is partitioned into:
  - Secular **precession** (~25 700 yrs): Sun (or Moon) attraction on Earth equatorial buldge, which is not in the ecliptic plane => torque that tends to bring the equator in the ecliptic plane (opposed by the centrifugal force due to Earth's rotation)
  - Periodic **nutation** (main period 18.6 yrs): irregularities of Earth's rotation around Sun and Moon around Earth => small oscillations (also called ***forced*** *nutations*)

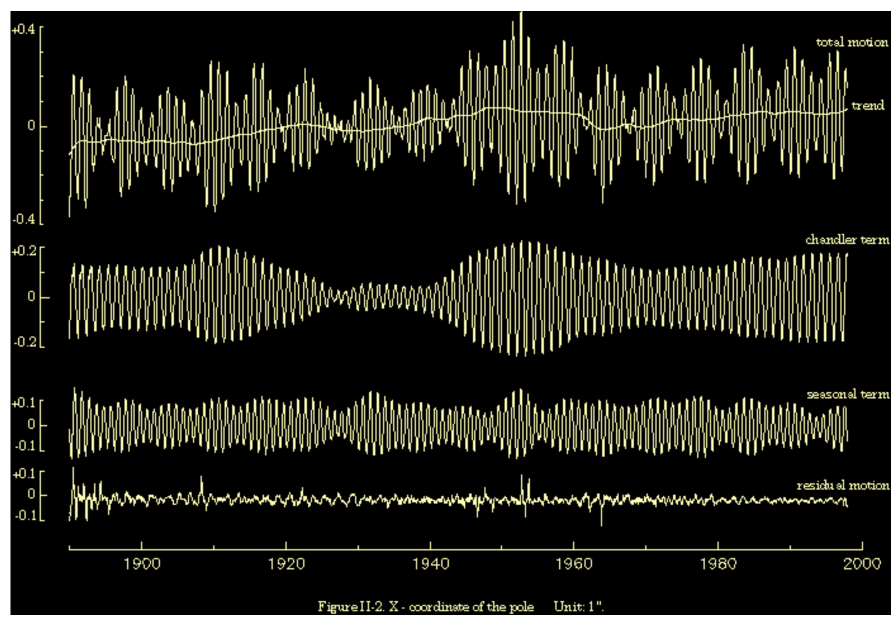


Space-fixed coordinate system/ Polar motion

- Because of mass redistributions in the earth (solid + atmosphere), the Earth's rotation axis moves with respect to Earth's crust: **polar motion**
- Three major components:
  - Chandler wobble (free oscillation) = period 435 days, 0.4" ~ 6 m amplitude
  - Annual oscillation forced by seasonal displacement of air and water masses, 0.15" ~ 2 m
  - Diurnal and semi-diurnal variations forced by oceanic tides ~ 0.5 m amplitude
- Non-oscillatory motion with respect to Earth's crust: polar wander, 3.7 mas/yr towards Groenland
- http://hpiers.obspm.fr/eop-pc/earthor/polmot/pm.html



Polar motion for the period 1996-2000.5 (dotted line) and polar wander since 1890 (doc. IERS)



(doc. IERS)

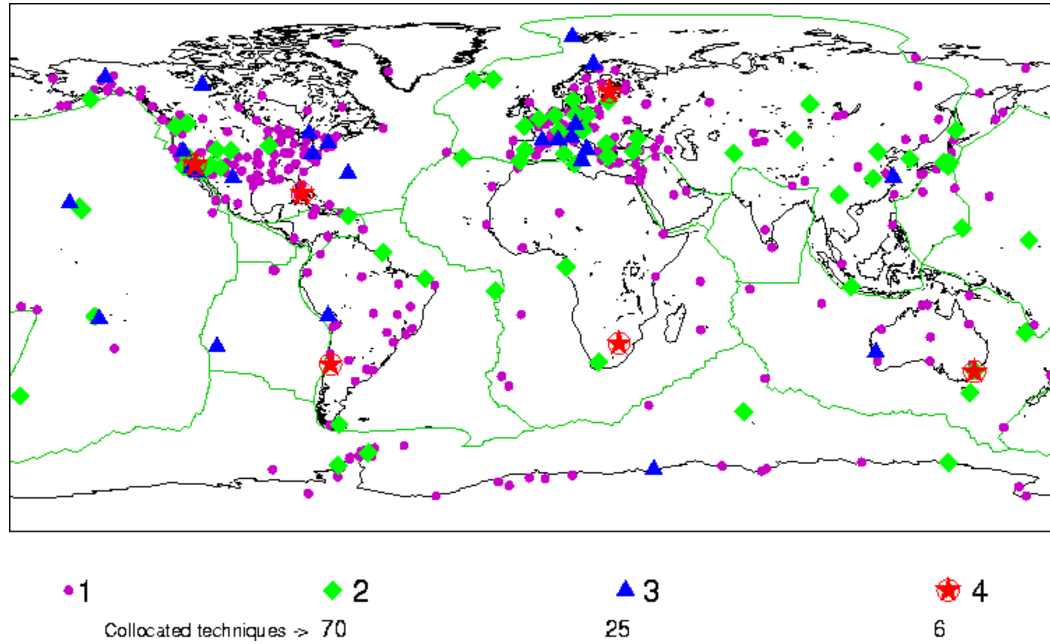
Conventional Systems

- Conventional **Inertial System**:
  - Orthogonal system, center = Earth center of mass, defined at standard epoch J2000 (January 1st, 2000, 12:00 UT)
  - Z = position of the Earth's angular momentum axis at standard epoch J2000
  - X = points to the vernal equinox
  - It is materialized by precise equatorial coordinates of extragalactic radio sources observed in Very Long Baseline Interferometry (VLBI) = Inertial Reference **Frame**.
  - First realization of the International Celestial Reference Frame (ICRF) in 1995.
- Conventional **Terrestrial System**:
  - Orthogonal system center = Earth center of mass
  - Z = position of the Earth's angular momentum axis at standard epoch J2000
  - X = Greenwich meridian
  - It is materialized by a set of ground control stations of precisely know positions and velocities = Terrestrial Reference **Frame**
- Systems are defined and frames are realized in the framework of an international service: the International Earth Rotation Service, IERS (http://www.iers.org/)
  - ➢ Mission of the IERS: *"To provide to the worldwide scientific and technical community reference values for Earth orientation parameters and reference realizations of internationally accepted celestial and terrestrial reference systems"*

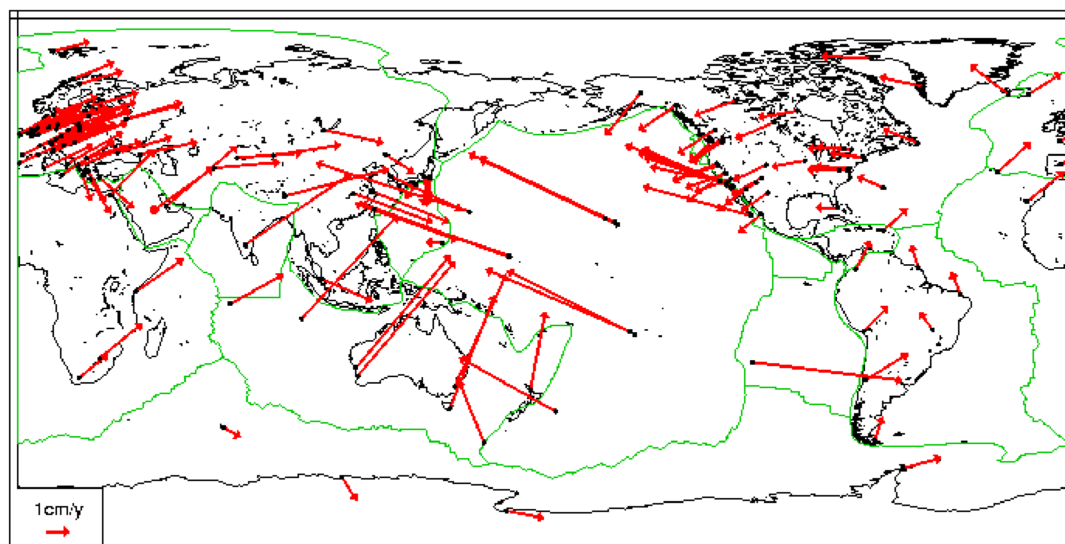
Reference systems/ Coordinate systems/ Terrestrial Reference Frame

- Realized by a set of ground control stations, in the framework the IERS
- International Terrestrial Reference Frame (http://lareg.ensg.ign.fr/ITRF):
  - First version in 1989 (ITRF-89), current version ITRF-2000
  - Set of station positions, velocities, and epochs in an Earth centered-Earth fixed (=ECEF) Terrestrial System
  - And associated variance-covariance matrix
  - A note on velocities:
    - Necessary because of plate motions!
    - Expressed in a no-net-rotation frame
  - Since ITRF derives from measurements, it changes (improves) with its successive realizations

Map of ground geodetic stations used in the definition of the ITRF-2000.
Note that some stations benefit from several observation techniques in colocation



ITRF97 Velocities (σ better than 3 mm/y)



Time systems/ Astronomic time scales

Astronomic time scales based on Earth rotation:

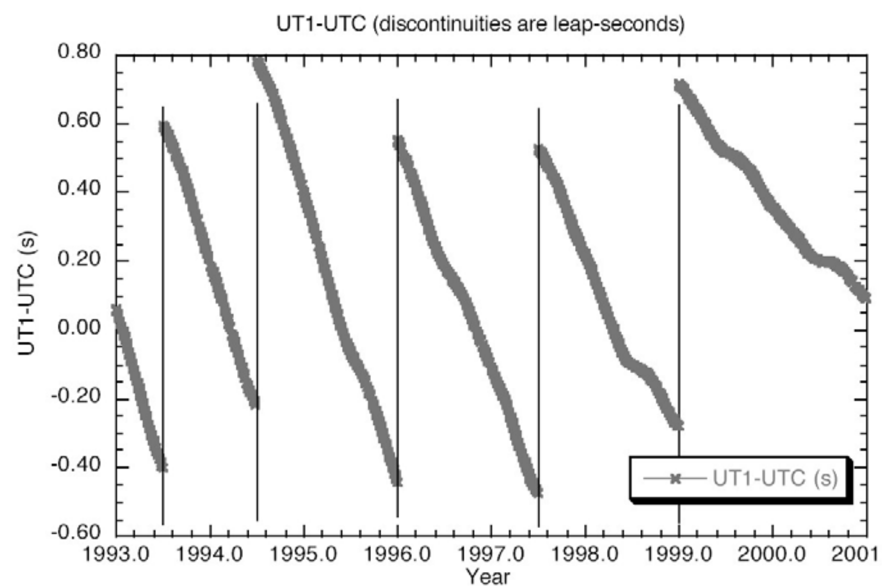
- **Universal Time UT**: hour angle of the Greenwich meridian
- **Sidereal Time ST**: hour angle of the vernal equinox (1 sidereal day = 23h56mn)
- UT and ST are not uniform because the Earth angular velocity varies (because of earth tides (< 2.5 ms), oceanic tides (< 0.03 ms), atmospheric circulation, internal effects, and transfer of angular momentum to the Moon orbital motion) => **UT corrected for polar motion is called UT1**

Time systems/Atomic time scales

Atomic time scales = practical realizations of time:

- **International Atomic Time TAI**: defined by its unit, the atomic second (at sea level)= *"Duration of 9,192,631,770 periods of the radiation corresponding to the transition between two hyperfine levels of the ground state of the Cesium-133 atom."*
- TAI not related to Earth rotation, but need for a relation! => **UT coordinated time UTC**:
  - Unit = atomic second
  - |UT1 – UTC| < 0.9 s
  - UTC changed in steps of 1 full second (leap second) if |UT1 – UTC| > 0.9 s, responsibility of the IERS (June 30 or Dec. 31)
  - UTC = broadcast time used for most civilian applications (your watch!)
  - |UT1-UTC| must be measured
- GPS Time GPST:
  - GPST = TAI – 19 sec
  - Coincident with UTC on January 6th, 1980, 00:00 UTC

Variations of UT1-TAI for 1994-1999



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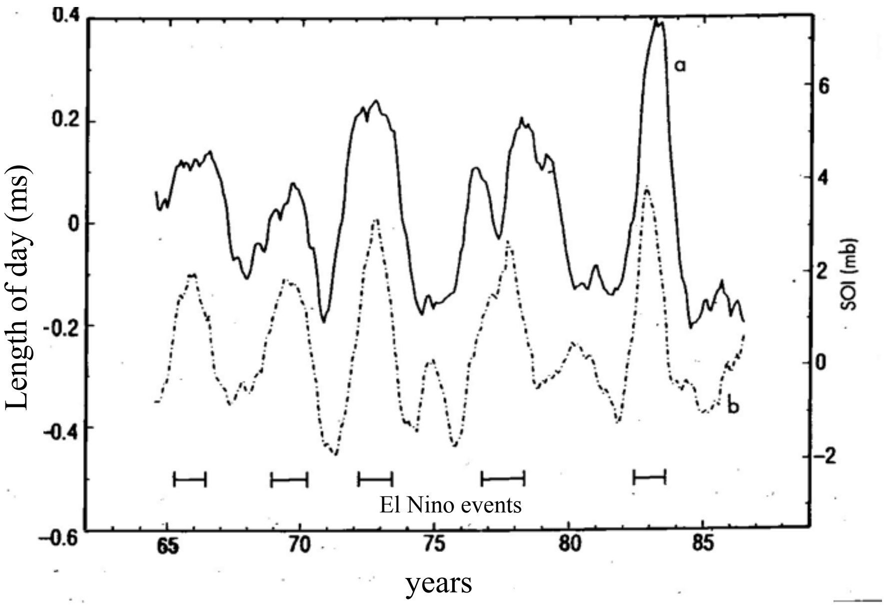
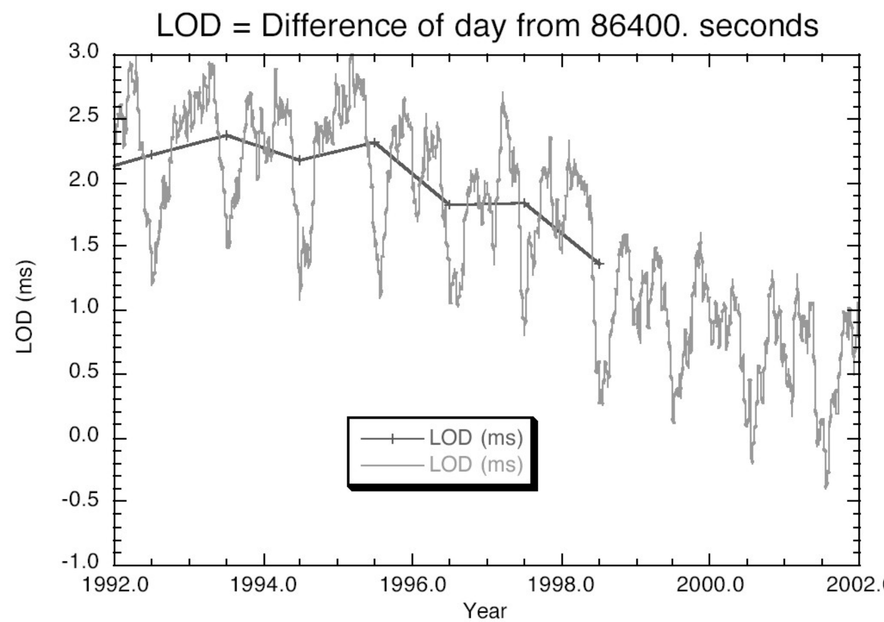
Time systems/ Length-of-day = LOD

- Length of Day (LOD) = difference between 86 400 sec SI and length of astronomical day:
  - Long term variations, cf. lunar tidal friction
  - Short term variations



Variations of LOD for 1965-1985, correlation with the Southern Oscillation Index (barometric pressure difference between Darwin and Tahiti)



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Time systems/Calendar

- **Julian Date** (JD): number of mean solar days elapsed since January 1st, 4713 B.C., 12:00
- **Modified Julian Date** (MJD) = JD – 2,400,00.5
  - Ex.: GPS standard epoch, JD = 2,444,244.5 (January 6th, 1980, 00:00 UTC)
  - Ex.: Standard epoch J2000.0, JD = 2,451,545.0 (January 1st, 2000, 12:00 UTC)
- **Day Of Year** (DOY): day since January 1st of the current year
- GPS calendar:
  - GPS week: Week since GPS standard epoch
  - GPS day of week: Sunday = 0 to Saturday = 6
  - GPS second: Second since GPS standard epoch

Summary

- The Earth is essentially an ellipsoid
- Its gravity field varies in space (and time)
- Its shape changes with time because of tides
- Its orientation with respect to a well-defined inertial system (materialized by stars, quasars) is known at all times if we know precession/nutation and UT1
- The position of the Earth's rotation axis with respect to the a well-defined terrestrial system (materialized by ground geodetic stations) is known at all times if we know polar motion
- Now we need conversion parameters between terrestrial and inertial systems.

Earth Orientation Parameters (EOP)

- The Earth's orientation is defined as the **rotation between**:
  - A **rotating geocentric set of axes linked to the Earth** (the terrestrial system materialized by the coordinates of observing stations) = Terrestrial System
  - A **non-rotating geocentric set of axes linked to inertial space** (the celestial system materialized by coordinates of stars, quasars, objects of the solar system) = Inertial System
- The transformation from Inertial Space to Terrestrial Frame can be made using a standard matrix rotation:

$$x_i = P \quad N \quad S \quad W \quad x_t$$

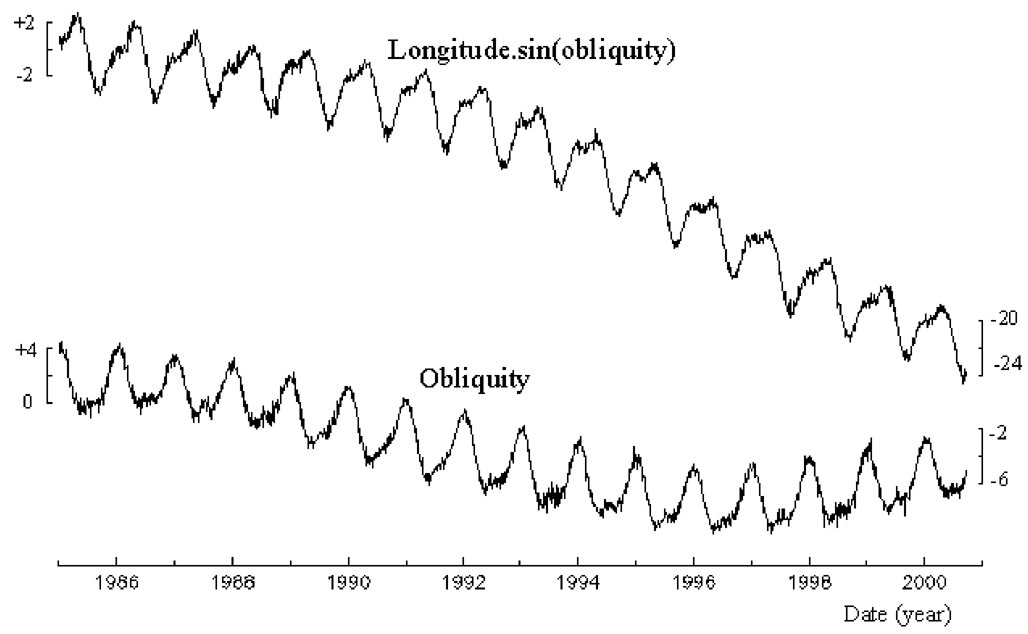
inertial (x_i), precession (P), nutation (N), spin (S), Polar motion (W), terrestrial (x_t)

Earth Orientation Parameters (EOP)

- Rotation parameters are given in astronomical tables provided by the IERS: for the most precise positioning, they are used as a priori values and sometimes adjusted in the data inversion.
- In practice, the IERS provides **five Earth Orientation Parameters** (EOP) :
  - Celestial pole offsets (dPsi, dEps): corrections to a given precession and nutation model
  - Universal time (UT1) = UT corrected for polar motion, provided as UT1-TAI
  - 2 coordinates of the pole (x,-y) of the CEP with respect to Earth's geographic pole axis = polar motion

Earth Orientation Parameters (EOP)

- Example of EOP tables



Celestial pole offsets: motion of the celestial pole relative to the IAU 1980 Theory of Nutation ant the IAU 1976 Precession. Unit:0.001"

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