# Accuracy Analysis of Determination the Vertical Displacements in Unstable Reference System

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**Keywords**: Unstable Reference System, Vertical Displacement, Foundation Plate Measurements

## SUMMARY

Measurements of horizontal and vertical displacements are carried out mostly based on reference points identified as fixed. In surveying practice there may be some situations in which there is not possible to perform the measurements with reference to stable points or the difficulties in identification of points' stability may arise. Accuracy analysis based on the covariance matrices have a special role in this process. In presented paper the covariance matrices were determined. The authors took advantage of the method of determination the vertical displacements in the absence of stability of reference points. The paper presents algorithm of computation and also results of the empirical tests. Tests were carried out based on simulated data of foundation plate measurements. The conclusions, which came from the calculations, encourage not only to do more detailed analysis and tests but also to theoretical development of the presented method.

## SUMMARY

Wyznaczanie przemieszczeń poziomych i pionowych wykonywane jest najczęściej w oparciu o zidentyfikowane jako stałe punkty referencyjne. Jednak w praktyce geodezyjnej mogą pojawić się sytuacje, w których nie będzie można wykonać pomiarów w nawiązaniu do uznanych za stabilne punktów referencyjnych lub identyfikacja stabilności punktów referencyjnych będzie bardzo utrudniona. Szczególną rolę w tym procesie odgrywa analiza dokładności wykorzystująca między innymi macierze kowariancji. W niniejszej pracy wyznaczono macierze kowariancji pewnej znanej z literatury przedmiotu metody wyznaczania przemieszczeń pionowych w przypadku braku stabilności punktów referencyjnych. Zaprezentowano także algorytm obliczeń przemieszczeń poszczególnych punktów oraz rezultaty testów empirycznych tej koncepcji przeprowadzonych na przykładzie symulowanych wyników pomiaru płyty fundamentowej. Uzyskane z obliczeń wnioski zachęcają do dalszych bardziej szczegółowych analiz i testów, a także teoretycznego rozwinięcia zaprezentowanej metody.

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## 1. INTRODUCTION

Currently the phenomenon of engineering objects' displacements is a common problem of interests because of the occurrence possibility, among others, construction disaster. The displacements may be caused by building for example: underground car parks in the center of the city or near the natural and artificial escarpments, tunnels under the rivers or the mountains, engineering objects' located in areas of unfavorable geotechnical conditions, etc. All these and other similar investments usually cause changes of the groundwater levels and thus the changes of geological properties. The consequence of these changes are, among others, the displacements of objects during the building and displacements of other engineering objects located near this investment. The surveying measurements associated with the determination of displacements are usually performed on the basis of reference points identified as fixed, eg. [Chen 1983, Caspary 2000]. The problem of identifying of the stability of the reference points is presented, among others in [Chen et al., 1990, Chrzanowski 2006 Prószynski, Kwaśniak 2006]. However, sometimes there is not possible to perform the measurements with reference to stable reference points or identification of such points is difficult. In this paper, the problem of determining the displacements is presented in reference to the situation where there is no reference points, and thus, the surveying measurements was carried out in an unstable reference system. This paper is a continuation of the considerations about the determination of the objects displacement in unstable reference system, what was set out in [Wiśniewski 1989]. The concept of determination of vertical objects displacements, which was presented there, lets us to generalize the problem to 3D system. What is more it was cause the search of the new solutions of this problem. Generalization of the problem connected with determination of the displacements in 3D unstable reference system was presented, among others, in papers [Kamiński 2008, 2008a, 2009, 2011]. In the search for new solutions of this problem, one of the paper is proposition [Zienkiewicz 2014]. The author used there the rules $M_{split}$ - estimation [Wiśniewski 2009, 2009a, 2009b]. As it was mentioned above, in paper [Wiśniewski 1989] the author proposed the concept of determination the vertical displacements in the case of absence of stability of reference points. In this paper, the algorithm of computations of displacements of individual points was proposed and the tests of practical application using simulated network of precise leveling were also carried out. Presented algorithm is using this conception. The accuracy analysis of the results of calculation was also performed.

## 2. THEORETICAL BASIS

The basic assumptions of the concept [Wiśniewski 1989] which were generalized to the three-dimensional systems, as it was mentioned above, were also presented in [Kamiński 2008, 2008a, 2009, 2011]. Therefore, in this paper only the most important and necessary in considerations theoretical assumptions of the method are presented. Assume that the considerations are carried out for small limited area, eg.: foundation plates located in factory halls or in warehouses, marine waterfront, airport runways, bridges, viaducts, etc. The geodetic measurements and their adjustment in two epochs are necessary for determining displacements, including vertical displacements. The first measurement is the initial measurement (epoch $t^{j=0}$). The second one is an actual measurement (epoch $t^{j=1}$). The theory of elaboration the results of this method does not use the reference points. This is the reason why the observations' adjustment is carry out as a free adjustment. During the estimation of free leveling network, the temporary and deformed surface of the object, which is displaced, is mapped to instantaneous optimal plane. The theoretical assumption of the method is free adjustment with estimation which takes into account the component parameters of vertical displacement: the angles of rotation around X axes $\varepsilon_{\mathrm{X}}^{j}$ and Y axes $\varepsilon_{\mathrm{Y}}^{j}$. There is no vector of translation ΔZ, because of absence of possibility of reference points identification. Thus it is not possible to determine the origin of local height systems' ($Z^{j}$=0) in both measurement epochs. It is worth noting that the parameters which are used to determine the vertical and horizontal displacements of the points (objects) are vectors of translation: ΔX, ΔY, ΔZ and three rotations around X, Y, Z axes by the angle value $\varepsilon_{\mathrm{X}}^{j}, \varepsilon_{\mathrm{Y}}^{j}, \varepsilon_{\mathrm{Z}}^{j}$, respectively. Based on papers, which are cited above, the free adjustment equation of correction, with taking into account the rotation parameters, for height difference between two points can be written as:

$$\mathrm{v}_{lk} = d_{k}^{j} - d_{l}^{j} - (\mathrm{Y}_{k} - \mathrm{Y}_{l})\varepsilon_{\mathrm{X}}^{j} + (\mathrm{X}_{k} - \mathrm{X}_{l})\varepsilon_{\mathrm{Y}}^{j} - h_{lk}^{j} \tag{1}$$

In the equation (1), the following designations are use:
$\mathrm{v}_{lk}$ – correction to measurement result (*l*, *k* – the designations of the controlled points, *l*, *k*=1,.., *r*, *r* – number of controlled points),
$h_{lk}^{j}$ – results of measurement in $t^{j}$ epoch, *j*=0, 1, … , (eg.: *j*=0 the initial measurement, *j*=1 the first actual measurement, etc.),
$d_{k}^{j},\ d_{l}^{j}$ – distances of temporary object's surface from instantaneous optimal plane in the moments of time $t^{j}$,
$\mathrm{Y}_{k}, \mathrm{Y}_{l}$ – Y coordinates of the controlled points *k*, *l*,
$\mathrm{X}_{k}, \mathrm{X}_{l}$ – X coordinates of the controlled points *k*, *l*,
$\varepsilon_{\mathrm{X}}^{j}$, $\varepsilon_{\mathrm{Y}}^{j}$ – angles of rotation around X and Y axes in epoch $t^{j}$, respectively

The equation of correction (1) can be presented in following matrix form

$$\mathbf{v} = \mathbf{A}\mathbf{d}^j + \mathbf{B}\boldsymbol{\varepsilon}^j - \mathbf{h}^j \tag{2}$$

where:
$\mathbf{v}$ – vector of corrections to results of observations,
$\mathbf{A}$, $\mathbf{B}$ – known matrices of coefficients,
$\mathbf{h}^j$ – vectors of free terms (results of measurement),
$\mathbf{d}^j = [d_1^j, \ldots, d_r^j]^{\mathrm{T}}$ – vector of distances of deformed surface from optimal plane in the moment $t^j$,
$\boldsymbol{\varepsilon}^j = [\varepsilon_{\mathrm{X}}^j, \varepsilon_{\mathrm{Y}}^j]^{\mathrm{T}}$ – vector of inclination angles of optimal plane in the moment $t^j$,
As it was mentioned above solution of the problem consists in realizing of free optimization in such a way, in which

$$\left.\begin{array}{c} \varphi\left(\mathbf{d}^j, \boldsymbol{\varepsilon}^j\right) = \mathbf{v}^{\mathrm{T}}\mathbf{P}\mathbf{v} = \min \\ \Phi\left(\mathbf{d}^j, \boldsymbol{\varepsilon}^j\right) = (\mathbf{d}^j)^{\mathrm{T}}\mathbf{P}_{\mathbf{d}}\mathbf{d}^j = \min \end{array}\right\} \tag{3}$$

($\mathbf{P}$ i $\mathbf{P}_{\mathbf{d}}$ – weight matrices, respectively: $\mathbf{P}$ – to measurement results $h_{lk}^j$ and $\mathbf{P}_{\mathbf{d}}$ – to possibly existing (previous) the accuracy assessment of the values $\mathbf{d}^j$)

The first equation of system of equations (3) refers to corrections to measurement result.
The second equation causes fitting of the instantaneous optimal plane in the deformed surface defined by the measured network.
The adjustment task of presented problem can be written in the form (assuming that: $\mathbf{P}_{\mathbf{d}} = \mathbf{I}_n$; $\mathbf{I}_n$ – identity matrix)

$$\left.\begin{array}{c} \mathbf{v} = \mathbf{A}\mathbf{d}^j + \mathbf{B}\boldsymbol{\varepsilon}^j - \mathbf{h}^j \\ \phi\left(\mathbf{d}^j, \boldsymbol{\varepsilon}^j\right) = \mathbf{v}^{\mathrm{T}}\mathbf{P}\mathbf{v} = \min \\ \psi\left(\mathbf{d}^j, \boldsymbol{\varepsilon}^j\right) = (\mathbf{d}^j)^{\mathrm{T}}\mathbf{d}^j = \min \end{array}\right\} \tag{4}$$

The solution of the system of equations (4) are vectors: $\boldsymbol{\varepsilon}^j$ of rotations $\varepsilon_{\mathrm{X}}^j, \varepsilon_{\mathrm{Y}}^j$, $\mathbf{d}^j$ distances and corrections $\mathbf{v}$ to observations, which have the following form:

$$\boldsymbol{\varepsilon}^j = [\mathbf{N}_3^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{N}_3]^{-1}\mathbf{N}_3^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{L}_{\mathbf{h}} \tag{5}$$

$$\mathbf{d}^j = -\mathbf{N}^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}(\mathbf{N}_3\boldsymbol{\varepsilon}^j - \mathbf{L}_{\mathbf{h}}) \tag{6}$$

$$\mathbf{v} = \mathbf{A}\mathbf{d}^j + \mathbf{B}\boldsymbol{\varepsilon}^j - \mathbf{h}^j \tag{7}$$

In the relationships (5), (6), (7) the following designations were used: $\mathbf{N} = [\mathbf{N}_1 \;\vdots\; \mathbf{N}_2]$, $\mathbf{N}_1 = \mathbf{A}_1^{\mathrm{T}}\mathbf{P}\mathbf{A}_1$, $\mathbf{N}_2 = \mathbf{A}_1^{\mathrm{T}}\mathbf{P}\mathbf{A}_2$, $\mathbf{N}_3 = \mathbf{A}_1^{\mathrm{T}}\mathbf{P}\mathbf{B}$, $\mathbf{L}_{\mathbf{h}} = \mathbf{A}_1^{\mathrm{T}}\mathbf{P}\mathbf{h}^j$. The block form of the matrix $\mathbf{A} = [\mathbf{A}_1 \;\vdots\; \mathbf{A}_2]$ is a result of necessity of using in computations the generalized inverses, which require the removal of the defect in free network, eg. [Caspary 2000,Wiśniewski 2009c].

The aim of the paper is to determine the vertical displacements. Therefore, the differences $\Delta\boldsymbol{\varepsilon}$ between the values of angles of rotation, obtained from solutions $\boldsymbol{\varepsilon}^{j+1}$ and $\boldsymbol{\varepsilon}^{j}$, are interesting.

$$\boldsymbol{\varepsilon}^{j+1} - \boldsymbol{\varepsilon}^{j} = \Delta\boldsymbol{\varepsilon} \tag{8}$$

The results obtained from the equation (8) suggest only the displacements of controlled points situated on research object. To determine the vertical displacements, several ways of further proceedings may be used. In this paper, the authors propose the following algorithm:

1. Checking if there are a reasons which allow us to formulate the conclusion about existing vertical displacements (verification based on: $\boldsymbol{\varepsilon}^{j+1} - \boldsymbol{\varepsilon}^{j} = \Delta\boldsymbol{\varepsilon}$).
2. Choosing the potentially stable point (non displaced) in both measurement epochs, based on adjustment results and accuracy analysis.
3. Adoption the height of stable point (eg. Z=0,0000 m).
4. Determination of height of all controlled points in both measurement epochs.
5. Determination of vertical displacements as a differences between heights of controlled points.
6. Carry out the tests of statistical significance of obtained vertical displacements using eg. Fisher distribution (*F* – test), [Caspary 2000].

## 3. ACCURACY ANALYSIS

An important role in process of determination the displacements has the accuracy analysis, which is represented by covariance matrices **C**. To determine the covariance matrix we use the covariance propagation law, which is well-known in adjustment calculus, and has the form (**D** – transformation matrix):

$$\mathbf{C} = \mathbf{D}\mathbf{C}_{\mathbf{h}}\mathbf{D}^{\mathrm{T}} \tag{9}$$

If we want to determine the covariance matrix $\mathbf{C}_{\boldsymbol{\varepsilon}}$ of vector $\boldsymbol{\varepsilon}$ ($\boldsymbol{\varepsilon} = \underbrace{[\mathbf{N}_3^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{N}_3]^{-1}\mathbf{N}_3^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{A}_1^{\mathrm{T}}\mathbf{P}}_{\mathbf{D}}\mathbf{h}$) , we obtain (for $\mathbf{C}_{\mathbf{h}} = \mathrm{m}_0^2\mathbf{P}^{-1}$)

$$\mathbf{C}_{\boldsymbol{\varepsilon}} = m_0^2\mathbf{D}\mathbf{P}^{-1}\mathbf{D}^{\mathrm{T}} \tag{10}$$

Thus, after performing the appropriate steps, the covariance matrix $\mathbf{C}_{\boldsymbol{\varepsilon}}$ has the form:

$$\mathbf{C}_{\boldsymbol{\varepsilon}} = m_0^2[\mathbf{N}_3^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{N}_3]^{-1}\mathbf{N}_3^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{N}_1(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{N}_3[\mathbf{N}_3^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{N}_3]^{-1} \tag{11}$$

in which the cofactors matrix (approximations of variance) $\mathbf{Q}_{\boldsymbol{\varepsilon}}$ can be written as follow:

$$\mathbf{Q}_{\boldsymbol{\varepsilon}} = [\mathbf{N}_3^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{N}_3]^{-1}\mathbf{N}_3^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{N}_1(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{N}_3[\mathbf{N}_3^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{N}_3]^{-1} \tag{12}$$

Coefficient: $m_0^2 = \frac{\mathbf{v}^{\mathrm{T}}\mathbf{P}\mathbf{v}}{n-r-de}$ ($n$ – number of observations, $r$ – number of unknowns , $de$ – defect of network)

The covariance matrix of the vector of corrections $\mathbf{C_v}$ has the form:

$$\mathbf{C_v} = m_0^2\mathbf{D}\mathbf{P}^{-1}\mathbf{D}^{\mathrm{T}} \tag{13}$$

where:

$$\mathbf{D} = [-\mathbf{A}\mathbf{N}^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{N}_3\{\mathbf{N}_3^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{N}_3\}^{-1}\mathbf{N}_3^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{A}_1^{\mathrm{T}}\mathbf{C_h}^{-1} + \mathbf{A}\mathbf{N}^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{A}_1^{\mathrm{T}}\mathbf{C_h}^{-1} + \\ +\mathbf{B}\{\mathbf{N}_3^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{N}_3\}^{-1}\mathbf{N}_3^{\mathrm{T}}(\mathbf{N}\mathbf{N}^{\mathrm{T}})^{-1}\mathbf{A}_1^{\mathrm{T}}\mathbf{C_h}^{-1} - \mathbf{I}_{\mathrm{n}}]$$

## 4. EXAMPLE OF PRACTICAL APPLICATION

Presented assumptions in theoretical part were verify on the example of simulated data, obtained from the measurement of foundation plate using method of precise leveling. The simulated network, measured height differences and location of controlled points are presented in the figure 1. It was assumed that measurements were performed to nine controlled points, which were regulary situated on the research plate. For simplifying the interpretation of the computation it was assumed that the heights of points are the same $Z_i$ = 0,0000m ($i$ =1,…,9).

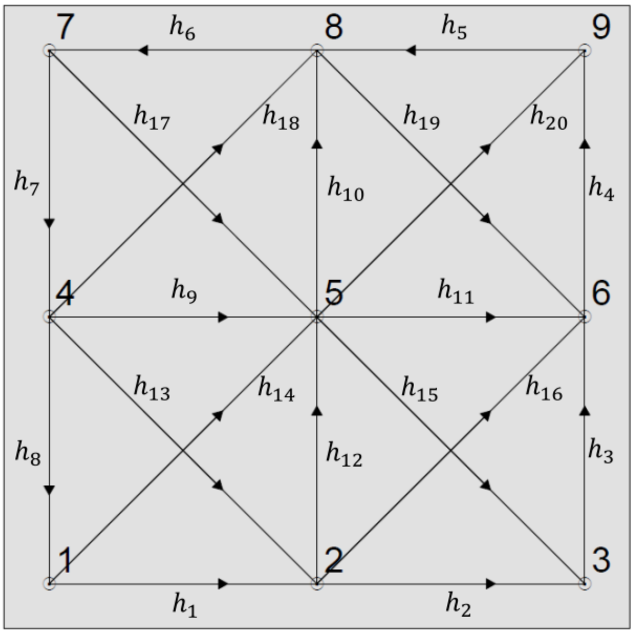


Fig. 1. Levelling control network

When simulating the measurement results it was assumed that the $X_i$ and $Y_i$ coordinates of all controlled points (Table 1) were determined using GPS RTK technology.

Table 1. Coordinates X and Y of controlled points.

| Point no. | X[m] | Y[m] | Point no. | X[m] | Y[m] | Point no. | X[m] | Y[m] |
|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 4 | 20 | 0 | 7 | 40 | 0 |
| 2 | 0 | 20 | 5 | 20 | 20 | 8 | 40 | 20 |
| 3 | 0 | 40 | 6 | 20 | 40 | 9 | 40 | 40 |

The displacement of the object is possible to determine when we have observations from at least two measurement epochs. In this paper, the initial measurement is associated with observations which were performed in epoch $t^{j=0}$, whereas the actual measurement is associated with observations which were performed in epoch $t^{j=1}$. The computation were divided into three variants. The differences between the particular variants of computations are connected with the simulated results in $t^{j=1}$ epoch.

Variant 1: It was assumed that the subsidence value of point no. 3 is 0,005m.
Variant 2: It was assumed that the subsidence value of points no. 3, 6 and 9 is 0,005m.
Variant 3: It was assumed that the subsidence value of points no. 2, 3 and 6 is 0,005m.
Simulated observations (height differences), which were used in further analysis, are presented in Table 2.

Table 2. Simulated observations in epochs $t^{j=0}$ and $t^{j=1}$.

| Height difference $h^j$ | Initial measurement | Actual measurement | | |
|---|---|---|---|---|
| | | Variant 1 | Variant 2 | Variant 3 |
| | epoch $t^{j=0}$ [m] | epoch $t^{j=1}$ [m] | epoch $t^{j=1}$ [m] | epoch $t^{j=1}$ [m] |
| $h_1$ | 0,0014 | 0,0014 | 0,0014 | -0,0036 |
| $h_2$ | 0,0009 | -0,0041 | -0,0041 | 0,0009 |
| $h_3$ | 0,0012 | 0,0062 | 0,0012 | 0,0012 |
| $h_4$ | 0,0011 | 0,0011 | 0,0011 | 0,0061 |
| $h_5$ | 0,0008 | 0,0008 | 0,0058 | 0,0008 |
| $h_6$ | 0,0007 | 0,0007 | 0,0007 | 0,0007 |
| $h_7$ | -0,0007 | -0,0007 | -0,0007 | -0,0007 |
| $h_8$ | -0,0008 | -0,0008 | -0,0008 | -0,0008 |
| $h_9$ | -0,0011 | -0,0011 | -0,0011 | -0,0011 |
| $h_{10}$ | -0,0012 | -0,0012 | -0,0012 | -0,0012 |
| $h_{11}$ | -0,0009 | -0,0009 | -0,0059 | -0,0059 |
| $h_{12}$ | -0,0014 | -0,0014 | -0,0014 | 0,0036 |
| $h_{13}$ | -0,0012 | -0,0012 | -0,0012 | -0,0062 |
| $h_{14}$ | 0,0012 | 0,0012 | 0,0012 | 0,0012 |
| $h_{15}$ | 0,0006 | -0,0044 | -0,0044 | -0,0044 |
| $h_{16}$ | -0,0006 | -0,0006 | -0,0056 | -0,0006 |
| $h_{17}$ | 0,0013 | 0,0013 | 0,0013 | 0,0013 |
| $h_{18}$ | -0,0013 | -0,0013 | -0,0013 | -0,0013 |
| $h_{19}$ | 0,0010 | 0,0010 | -0,0040 | -0,0040 |
| $h_{20}$ | -0,0010 | -0,0010 | -0,0060 | -0,0010 |

The following matrices were used during the computation:

$$\mathbf{A} = \left[ \underbrace{\begin{matrix} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \end{matrix}}_{\mathbf{A}_1} \;\vdots\; \underbrace{\begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{matrix}}_{\mathbf{A}_2} \right] \quad \mathbf{B} = \begin{bmatrix} -20 & 0 \\ -20 & 0 \\ 0 & 20 \\ 0 & 20 \\ 20 & 0 \\ 20 & 0 \\ 0 & -20 \\ 0 & -20 \\ -20 & 0 \\ 0 & 20 \\ -20 & 0 \\ 0 & 20 \\ -20 & -20 \\ -20 & 20 \\ -20 & -20 \\ -20 & 20 \\ -20 & -20 \\ -20 & 20 \\ -20 & -20 \\ -20 & 20 \end{bmatrix} \quad \mathbf{h}^{j=0} = \begin{bmatrix} 0{,}0014 \\ 0{,}0009 \\ 0{,}0012 \\ 0{,}0011 \\ 0{,}0008 \\ 0{,}0007 \\ -0{,}0007 \\ -0{,}0008 \\ -0{,}0011 \\ -0{,}0012 \\ -0{,}0009 \\ -0{,}0014 \\ -0{,}0012 \\ 0{,}0012 \\ 0{,}0006 \\ -0{,}0006 \\ 0{,}0013 \\ -0{,}0013 \\ 0{,}0010 \\ -0{,}0010 \end{bmatrix}$$

$\boldsymbol{\varepsilon}^j = [\varepsilon_X^j, \varepsilon_Y^j]^T$ and $\mathbf{d}^j = [d_1^j, d_2^j, d_3^j, d_4^j, d_5^j, d_6^j, d_7^j, d_8^j, d_9^j]^T$

Variant1:

$$\mathbf{h}^{j=1} = \begin{bmatrix} 0{,}0014 \\ -0{,}0041 \\ 0{,}0062 \\ 0{,}0011 \\ 0{,}0008 \\ 0{,}0007 \\ -0{,}0007 \\ -0{,}0008 \\ -0{,}0011 \\ -0{,}0012 \\ -0{,}0009 \\ -0{,}0014 \\ -0{,}0012 \\ 0{,}0012 \\ -0{,}0044 \\ -0{,}0006 \\ 0{,}0013 \\ -0{,}0013 \\ 0{,}0010 \\ -0{,}0010 \end{bmatrix}$$

Variant 2:

$$\mathbf{h}^{j=1} = \begin{bmatrix} 0{,}0014 \\ -0{,}0041 \\ 0{,}0012 \\ 0{,}0011 \\ 0{,}0058 \\ 0{,}0007 \\ -0{,}0007 \\ -0{,}0008 \\ -0{,}0011 \\ -0{,}0012 \\ -0{,}0059 \\ -0{,}0014 \\ -0{,}0012 \\ 0{,}0012 \\ -0{,}0044 \\ -0{,}0056 \\ 0{,}0013 \\ -0{,}0013 \\ -0{,}0040 \\ -0{,}0060 \end{bmatrix}$$

Variant 3:

$$\mathbf{h}^{j=1} = \begin{bmatrix} -0{,}0036 \\ 0{,}0009 \\ 0{,}0012 \\ 0{,}0061 \\ 0{,}0008 \\ 0{,}0007 \\ -0{,}0007 \\ -0{,}0008 \\ -0{,}0011 \\ -0{,}0012 \\ -0{,}0059 \\ 0{,}0036 \\ -0{,}0062 \\ 0{,}0012 \\ -0{,}0044 \\ -0{,}0006 \\ 0{,}0013 \\ -0{,}0013 \\ -0{,}0040 \\ -0{,}0010 \end{bmatrix}$$

The authors assume in computations the identity weights matrix **P** = Diag (1,…,1).

Conducting the computations according to the presented conception let us to obtain the solutions in three variants. The differences Δε between the values obtained from computations: $\varepsilon^{j=1}$ and $\varepsilon^{j=0}$ were also determined. Based on covariance matrices $\mathbf{C}_{\boldsymbol{\varepsilon}}$**,** the errors of estimated components of displacement $\varepsilon_{\mathrm{X}}^{j}, \varepsilon_{\mathrm{Y}}^{j}$ were determined by using the relationship $m_{\varepsilon} = \sqrt{(\mathbf{C}_{\boldsymbol{\varepsilon}})_{ii}}$ (where: the indexes $ii$ denote the diagonal elements of covariance matrix $\mathbf{C}_{\boldsymbol{\varepsilon}}$**,** (11))

Table 3. Results of adjustment for variant 1

| Angle of rotation | The value of angle of rotation in epoch $t^{j=0}$ [cc] | Mean error of angle of rotation $m_{\varepsilon}$ [cc] | The value of angle of rotation in epoch $t^{j=1}$ [cc] | Mean error of angle of rotation $m_{\varepsilon}$ [cc] | Δε [cc] |
|---|---|---|---|---|---|
| $\varepsilon_{\mathrm{X}}^{j}$ | 1,3 | 11,25 | 27,8 | 11,25 | 26,5 |
| $\varepsilon_{\mathrm{Y}}^{j}$ | -3,5 | 11,25 | 23,1 | 11,25 | 26,6 |

Table 4. Results of adjustment for variant 2

| Angle of rotation | The value of angle of rotation in epoch $t^{j=0}$ [cc] | Mean error of angle of rotation $m_{\varepsilon}$ [cc] | The value of angle of rotation in epoch $t^{j=1}$ [cc] | Mean error of angle of rotation $m_{\varepsilon}$ [cc] | Δε [cc] |
|---|---|---|---|---|---|
| $\varepsilon_{\mathrm{X}}^{j}$ | 1,3 | 11,25 | 80,8 | 11,25 | 79,5 |
| $\varepsilon_{\mathrm{Y}}^{j}$ | -3,5 | 11,25 | -3,5 | 11,25 | 0,0 |

Table 5. Results of adjustment for variant 3

| Angle of rotation | The value of angle of rotation in epoch $t^{j=0}$ [cc] | Mean error of angle of rotation $m_{\varepsilon}$ [cc] | The value of angle of rotation in epoch $t^{j=1}$ [cc] | Mean error of angle of rotation $m_{\varepsilon}$ [cc] | Δε [cc] |
|---|---|---|---|---|---|
| $\varepsilon_{\mathrm{X}}^{j}$ | 1,3 | 11,25 | 54,3 | 11,25 | 53,0 |
| $\varepsilon_{\mathrm{Y}}^{j}$ | -3,5 | 11,25 | 49,6 | 11,25 | 53,1 |

Analyzing the adjustment results presented in tables 3, 4 and 5, which are the components of the vector of vertical displacement, we note that in all analyzed variants exist the suspicion about the occurrence of vertical displacements. For further considerations, to realize the proposed algorithm in theoretical part, it was assumed that the height of point no. 1 is constant ($Z_1$ = 0,0000m). Then the heights of all controlled points were determined in both epochs. The obtained results are presented in table 6.

Table 6. Heights $Z_i$ of controlled points

| Controlled point no. | Heights of controlled points $Z_i$ in epoch $t^{j=0}$ [m] | Heights of controlled points $Z_i$ in epoch $t^{j=1}$. Variant 1 [m] | Heights of controlled points $Z_i$ in epoch $t^{j=1}$. Variant 2 [m] | Heights of controlled points $Z_i$ in epoch $t^{j=1}$. Variant 3 [m] |
|---|---|---|---|---|
| 1 | 0,00000 | 0,00000 | 0,00000 | 0,00000 |
| 2 | **0,00105** | 0,00105 | 0,00105 | **-0,00395** |
| 3 | **0,00099** | **-0,00401** | **-0,00401** | **-0,00401** |
| 4 | 0,00136 | 0,00136 | 0,00136 | 0,00136 |
| 5 | 0,00099 | 0,00099 | 0,00099 | 0,00099 |
| 6 | **0,00063** | 0,00063 | **-0,00437** | **-0,00437** |
| 7 | 0,00087 | 0,00087 | 0,00087 | 0,00087 |
| 8 | 0,00016 | 0,00016 | 0,00016 | 0,00016 |
| 9 | **0,00036** | 0,00036 | **-0,00464** | 0,00036 |

Based on the data presented in table 6 we can determine the vertical displacements ($p_i = Z_i^{j=1} - Z_i^{j=0}$, where $i$=1,2,…,9 – number of controlled points) and obtain:
Variant 1. Point no. 3, $p_3$ = – 0,00401 – 0,00099 = – 0,005m.
Variant 2. Point no. 3, $p_3$ = – 0,00401 – 0,00099 = – 0,005m
Point no. 6, $p_6$ = – 0,00437 – 0,00063 = – 0,005m
Point no. 9, $p_9$ = – 0,00464 – 0,00036 = – 0,005m.
Variant 3. Point no. 2, $p_2$ = – 0,00395 – 0,00105 = – 0,005m
Point no. 3, $p_3$ = – 0,00401 – 0,00099 = – 0,005m
Point no. 6, $p_6$ = – 0,00437 – 0,00063 = – 0,005m.
When commenting the results we can observed that the values of vertical displacements from adjustment correspond to adopted in example theoretical values. According to the fact that the simulated results of observation were taken into account, in this paper the $F$ - test connected with significance of obtained vertical displacements is not performed.

## 5. CONCLUSIONS

In this paper, the practical properties of conception of determination the vertical displacements in unstable reference systems were analized. The analysis were performed on simulated network of precise leveling. To determine the vertical displacements of controlled points the authors proposed also a simplified algorythm of calculation. The obtained results do not let us to draw the general conclusions. The presented proposition of determination the vertical displacements requires carrying out a lot of detailed theoretical and empirical analysis.

## REFERENCES


1. Caspary W. F., 2000, Concepts of network and deformation analysis, Monograph 11, School of Geomatic Engineering, The University of New South Wales, UNSW Sydney NSW 2052 Australia.
2. Chen Y. Q., 1983, Analysis of deformation surveys - a generalized method. Technical Report N0. 94, University of New Brunswick, Fredericton, pp. 54 – 72.
3. Chen Y. Q., Chrzanowski A. and Secord J.M., 1990, A strategy for the analysis of the stability of reference points in deformation surveys, CISM Journal ACSGC, 44(2), pp.141-149.
4. Chrzanowski A., 2006, Accuracy evaluation of geodetic monitoring of deformations in large open pit mines, 3rd IAG/12th FIG Symposium on Deformation Measurements, Baden, 22-25 May.
5. Kamiński W., 2008, The idea of monitoring surface deformations on unstable ground with the use of GPS technology, Bellotino di Geodesia e Scienze Affini, No 1, pp.33-45.
6. Kamiński W., 2008a, The conception of monitoring the superficial deformations located on the unstable foundation with the usage of GPS technology, 4th IGS Symposium on Geodesy for Geotechnical and Structural Engineering and 13th FIG Symposium on Deformation Measurements and Analysis, Lisbon, 12-15 May.
7. Kamiński W., 2009, Properties and analysis of the accuracy of estimation results obtained by the DiSTFA method in monitoring displacements and strains, Geodesy and Cartography, vol, 58, No 2, pp. 105-118.
8. Kamiński W., 2011, DiSTFAG method robust to gross errors in monitoring displacement and strains in unstable reference systems, Geodesy and Cartography, vol, 60, No 1, pp. 21-33.
9. Prószyński W., Kwaśniak M., 2006, Podstawy geodezyjnego wyznaczania przemieszczeń. Pojęcia i elementy metodyki, Wyd. Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa, (in Polish).
10. Wiśniewski Z., 1989, Koncepcja wyznaczania parametrów położenia i kształtu płyt fundamentowych na podstawie niwelacji swobodnej, VI sesja naukowo-techniczna, Aktualne problemy naukowe i techniczne prac geodezyjnych, Gdańsk-Sobieszewo, 6-7 października, (in Polish).
11. WiśniewskiZ., 2009, Estimation of parameters in a split functional model of geodetic observations ($M_{split}$ estimation), Journal of Geodesy vol. 83, pp. 105- 120, DOI: 10.1007/s00190-008-0241.
12. WiśniewskiZ., 2009a, $M_{split}$ estimation. Part I: Theoretical foundation, Geodesy and Cartography, 58(1), pp.3-21, 2009.
13. WiśniewskiZ., 2009b, $M_{split}$ estimation. Part II: Squared $M_{split}$ estimation. Numerical examples. Geodesy and Cartography, Vol. 58, No 1, str. 23-48.
14. Wiśniewski Z., 2009c, Rachunek Wyrównawczy w geodezji (z przykładami), Wyd. UWM w Olsztynie, Olsztyn, (in Polish).
15. Zienkiewicz M. H.. 2014, Application of $M_{split}$ estimation to determine control points displacements in networks with unstable reference system, Survey Review, DOI:10.1179/1752270614Y.0000000105

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